Queue Evolution on Freeways Leading to a Single Core City During the Morning Peak

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ABSTRACT

The benefits of urban highway projects derive mainly from reduction of delays in traffic queues. However, current static network analysis ignores the fact queues develop on a network. The objective of this research is to obtain the cumulative arrival curves at a finite number of bottlenecks around a single core city during the morning peak. It is necessary, therefore, to identify when and which bottleneck each commuter passes, given commuters' home location and desired arrival times (work schedules). Each commuter is assumed to have a common form of trip cost function which consists of a static cost of free flow travel time, a time-dependent congestion cost due to queueing delay in a queue, and schedule delay (the difference between his actual and desired arrival times). Commute trips are assigned spatially and temporally to bottlenecks, so as to establish an equilibrium in which each commuter seeks to minimize his trip cost. This analysis employs graphical queueing and spatial assignment techniques on a continuum demand space with a many-to-one origin-destination pattern. Some queueing patterns for a two-bottleneck geometry are analysed in detail.

1.0. INTRODUCTION

In conventional network analysis, peak traffic flow on a network is estimated by assigning a certain fraction of the daily traffic uniformly over a peak period. This leads to traffic flow patterns which are unrealistic in two major respects. First, the analysis is static, so it does not consider commuters' departure time choices. Second, it ignores the existence of queues on the network.

On a freeway, delays caused by queues are typically much larger than delays outside of queues. Indeed, the travel speed on a freeway is almost independent of the traffic flow until the flow is close to saturation. Furthermore, typical commute trip lengths are so short -- an average is less than ten miles per trip -- that any decrease in speed except in a queue

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does not create significant delay. Delays due to other minor road congestion also typically give a negligible contribution to individual trip costs, since flows on minor roads are likely to be well below capacity.

One effect of queues is that they cause schedule delay during the morning peak, defined as the difference between a commuter's actual and desired arrival times at work (Hurdle[6]). The cost due to this schedule delay should be included the network analysis.

Departure time choices at a single bottleneck have been analysed by several researchers such as Vickrey [12], Henderson [4], Hendrickson, et al. [5], Hurdle [7], Fargier [3], and DePalma, et al. [2]. They developed methods to estimate the cumulative arrival curve at a single bottleneck during the morning and evening peaks, given commuters' desired departure times from the bottleneck in the morning peak or arrival times at the bottleneck in the evening peak. Each commuter was assumed to pay a cost due to queueing and schedule delays. Commuters were then assigned temporally so as to establish an equilibrium in which no one can find a better choice than the one assigned.

The following research can be considered as an extension of the above single bottleneck analyses to a geometry such as in Fig.1-1 with more than one bottlenecks. In a single core city, a residential area ,where homes of commuters are distributed approximately continuously, surrounds a concentrated working place. The working place is so concentrated that the O-D(origin-destination) pattern can be treated as many-to-one (a point working place). The network consists of two types of roads: many finely spaced minor roads and a few major roads (freeways) leading to the working place. The function of the minor roads is to carry traffic between homes and the major roads; while the major roads gather traffic from the minor roads, bring it near the destination, and release it to other minor roads.

The major road network is typically not very complicated. It consists of a few "tree" type structures in which one tree may intersect another tree. Here the bottlenecks are assumed to be on the trunks of the trees possibly at junctions of the major roads. Each commuter is assumed to pass only one bottleneck on his way to the city core.

Our objective is to construct the cumulative arrival curves at bottlenecks during the morning peak so as to establish an equilibrium,

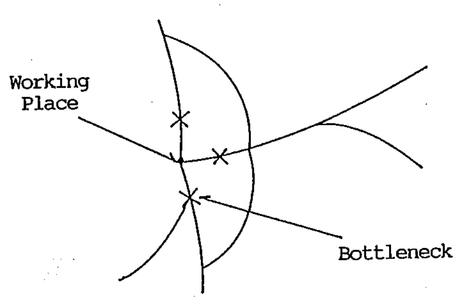

Fig.1-1. Tree Structures of a Major Road Network.

given commuters' work schedules and home locations. The problem involves both temporal and spatial assignments, that is, we must determine when and which bottleneck each commuter chooses.

1.1. AN OUTLINE OF THE PROCEDURE

Home locations and work schedules of commuters are represented in a three-dimensional space (Fig.1-2) with the horizontal plane showing home locations $x=(x_1,x_2)$ and the vertical axis showing work schedule time t_w. Each commuter can be represented as a point in this demand space. The set of all such points are assumed to be distributed approximately continuously over some region of this three-dimensional space with a rate density $\rho(x,t_w)$.

To determine a commuter's choice of departure time and bottleneck, a trip cost function is introduced. Each commuter is assumed to have a common form of the trip cost function which consists of a cost of free flow travel time and a bottleneck cost due to queueing and schedule delays. Since the travel speed on the network is assumed to be (nearly) independent of the flow, the cost of free flow travel time is static, depending on only a home location. The bottleneck cost is time-dependent, but it depends only on the bottleneck behavior (independent of home locations). These assumptions simplify the analysis tremendously, because, to update the trip cost temporally, we need only know the queueing and schedule delays at bottlenecks. We do not need to know the flows on every link of the network.

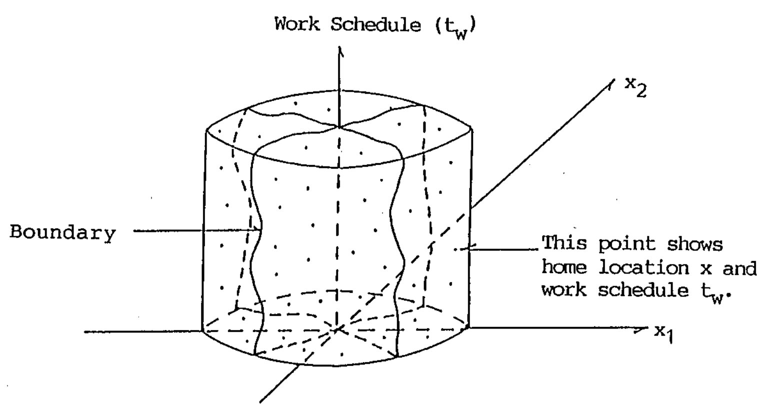

Fig. 1-2. The Three-Dimensional Demand Space.

We decompose the problem into two parts: spatial and temporal assignments. If we knew which bottleneck each commuter chose, the three-dimensional demand space could be partitioned into regions served by the same bottleneck, with boundaries as in Fig.1-2 (the spatial assignment). In each subset of the demand space, we recognize that the evaluation of departure times is the same as for a single bottleneck (the temporal assignment). Once we temporally assign points in each subset, we can then know when and which bottleneck each commuter uses. Thus, we can draw the cumulative arrival curves at all bottlenecks, and evaluate the trip costs for every commuter. Evaluation of trip cost for everyone, in turn, allows us to split the demand space into subsets.

Consequently, the structure of the problem forms a loop. As the following sections show, to close the loop is equivalent to solving two sets of differential equations simultaneously: one for the temporal assignment and another for the spatial assignment.

2.0. FORMULATION

For each bottleneck j, j=1,2,.....,J, let

$A_j(t)$ = cumulative number of commuters arriving at bottleneck j by time t,

$D_j(t)$ = cumulative number of commuters departing from bottleneck j by time t,

$W_j(t)$ = cumulative number of commuters passing bottleneck j whose work starting time is earlier than time t,

as illustrated in Fig.2-1.

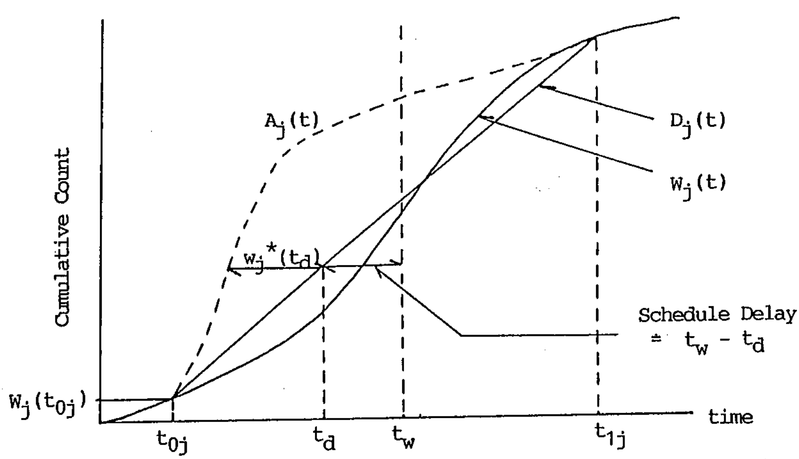

Fig.2-1. A Queueing Pattern in the Morning Peak at Bottleneck j.

Although these curves are presently unknown, we assume that there exists some times t_{0j} when a queue first forms at bottleneck j, times t_{1j} when the queue finally terminates, and that

$$A_j(t) = D_j(t) = W_j(t), \quad \text{for} \quad t \leq t_{0j},\ t_{1j} \leq t.$$

For $t_{0j} < t < t_{1j}$ when there is a positive queue, the flow through the bottleneck is assumed to have a given constant value $dD_j(t)/dt = \mu_j$. Commuters are served in order of their arrivals: FIFO (First In First Out), so the queueing delay $w_j^*(t_d)$ of a commuter who departs from bottleneck j at time t_d is the horizontal distance $t_d - A_j^{-1}(D_j(t_d))$ between the curves A_j and D_j.

If a commuter has home location x and work starting time t_w, we assume that he chooses bottleneck j and departure time from the bottleneck t_d, so as to minimize a trip cost function of the form (the same form for everyone),

$$TC_j^*(t_d \mid x, t_w) = f_1\{m_j(x)\} + f_2\{w_j^*(t_d)\} + f_3\{t_w - t_d\} \tag{2.1}$$

$$= f_1\{m_j(x)\} + p_j^*\{t_d, t_w\},$$

where:

$m_j(x)$ = travel time from home location x to the working place via bottleneck j,

$t_w - t_d$ = schedule delay for a commuter with work starting time t_w who departs from bottleneck j at time t_d,

$p_j^*\{t_d, t_w\} = f_2\{w_j^*(t_d)\} + f_3\{t_w - t_d\}$ = bottleneck cost.

The $f_1(m)$, $f_2(w)$, and $f_3(s)$ are specified functions. The travel cost function $f_1(m)$ and the queueing cost function $f_2(w)$ are both assumed to be monotone increasing and positive for $m > 0$ or $w > 0$. The cost $f_3(s)$ for schedule delay is assumed to be positive and convex with $f_3(0) = 0$. The schedule delay could be either positive or negative.

This trip cost function consists of three separable components. Travel time is the time consumed for a trip without congestion delay, obtained from the travel distance between a home and the working place, and the free flow travel speed. Travel speed is assumed to be independent of link flow, so the travel cost $f_1\{m_j(x)\}$ is static and a function only of the bottleneck j and home location x. Queueing delay depends on departure time t_d but is independent of the work schedule t_w, because of the FIFO queue discipline. Schedule delay can be considered to depend on departure time t_d and work schedule t_w.[3)] The bottleneck cost depends on the commuters work schedule t_w and the queueing situation at bottleneck j but is independent of his home location x.

2.1. TEMPORAL EQUILIBRIUM CONDITION

Since the bottleneck cost is the same for all commuters who choose the same bottleneck j (independent of x), the temporal equilibrium condition for those commuters who choose bottleneck j is the same as that described by Smith[11] and Daganzo[1] for a single bottleneck and a given work schedule curve $W_j(t)$. It is convenient here, however, to describe this condition in a somewhat different form than they gave.

Each commuter with work starting time t_w, knowing the bottleneck cost $p_j^*(t_d,t_w)$, would choose to pass the bottleneck at such time $t_d(t_w)$ as to minimize his trip cost. Thus, for given t_w, his t_d should satisfy

$$\partial TC_j^*(t_d \mid x,t_w)/\partial t_d = \partial p_j^*(t_d,t_w)/\partial t_d = 0 \ .$$

The derivative of the trip cost with respect to t_w is therefore

$$\begin{aligned} dp_j^*(t_d(t_w),t_w)/dt_w &= \partial p_j^*(t_d,t_w)/\partial t_w + \partial p_j^*(t_d,t_w)/\partial t_d \; dt_d(t_w)/dt_w \\ &= \partial p_j^*(t_d,t_w)/\partial t_w \ , \end{aligned} \tag{2.2}$$

since the second term is zero at his departure time $t_d = t_d(t_w)$. Thus, we obtain the temporal equilibrium condition:[1)]

$$p_j^{*\prime}(t_d(t_w),t_w) = \partial p_j^*(t_d(t_w),t_w)/\partial t_w = f_3'\{t_w - t_d(t_w)\}. \tag{2.3}$$

Smith [11] and Daganzo [1] also showed that commuters arrive and leave the queue in the same order as their work starting times, under the assumptions we have made. [2)] This we call the FIFW (First In First Work) discipline. Under this discipline, t_d and t_w are explicitly related to each other in that:

$$D_j(t_d) = W_j(t_w) \ , \qquad t_d = t_d(t_w) = D^{-1}_j(W_j(t_w)) \ . \tag{2.4}$$

Thus, the trip cost function can be expressed in terms of only x and t_w,

$$\begin{aligned} TC_j(x,t_w) &= f_1\{m_j(x)\} + f_2\{w_j(t_w)\} + f_3\{s_j(t_w)\} \\ &= f_1\{m_j(x)\} + p_j(t_w) \ , \end{aligned} \tag{2.5}$$

where:

$$TC_j(x,t_w) = TC_j^*(t_d(t_w) \mid x,t_w),$$

$$w_j(t_w) = w_j^*(t_d(t_w)),$$

$$s_j(t_w) = t_w - t_d(t_w),$$

$$p_j(t_w) = p_j^*(t_d(t_w),t_w) = f_2\{w_j(t_w)\} + f_3\{s_j(t_w)\}.$$

As a result, the temporal equilibrium conditions for bottleneck j is [4)]

$$p_j'(t) = f_3'\{s_j(t)\}. \tag{2.6}$$

If the schedule delay $s_j(t)$ were known, we could integrate this equation and evaluate the bottleneck cost

$$p_j(t) = \int_{t_{0j}}^{t} f_3'\{s_j(x)\}dx \,. \tag{2.7}$$

In most of the following illustrations, however, we will assume that $f_3(s)$ is piecewise linear in s,

$$f_3(s) = \begin{cases} c_1 s, & s \geq 0, \quad c_1 > 0, \\ -c_2 s, & s < 0, \quad c_2 > 0. \end{cases} \tag{2.8}$$

In this case Eq.(2.6) becomes

$$\begin{aligned} p_j'(t) &= c_1 \,, & s_j(t) > 0 \,, \\ -c_2 \leq p_j'(t) &\leq c_1 \,, & s_j(t) = 0 \,, \\ p_j'(t) &= -c_2 \,, & s_j(t) < 0 \,. \end{aligned} \tag{2.9}$$

Thus $p_j(t)$ increases or decreases at a rate independent of $s_j(t)$ as long as $s_j(t)$ remains positive or negative.

2.2. A SINGLE BOTTLENECK

If there is only one bottleneck or if the geometry of the network has some symmetry such as shown in Fig. 2-2 so that we knew ahead of time (by symmetry) which bottleneck each commuter chooses, then we would also know the total number of commuters from all locations x with work starting time before t who choose bottleneck j, $W_j(t)$.

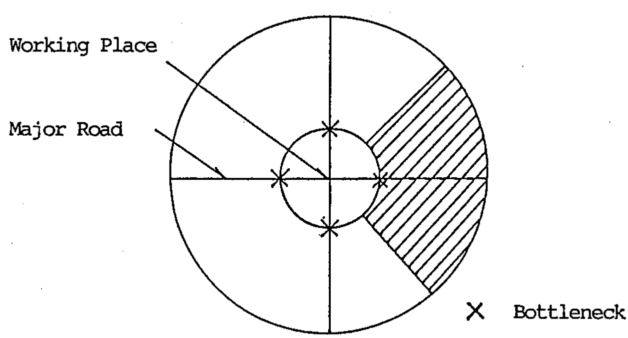

Fig. 2-2. A Symmetric Geometry.

If, with the known $W_j(t)$, we also knew the time t_{0j} when a queue formed, we could also draw the curve $D_j(t)$ as a straight line from $(t_{0j}, W_j(t_{0j}))$ with a slope μ_j as shown in Fig. 2-1. From the curves $W_j(t)$ and $D_j(t)$, we could also evaluate the schedule delay $s_j(t) = t - D^{-1}_j(W_j(t))$ for $t_{0j} < t \leq t_{1j}$. Equation (2.7) would then determine the bottleneck cost $p_j(t)$. Knowing the total bottleneck cost $p_j(t)$ and the cost of schedule delay, we can evaluate that part of the cost due to queueing delay and thus determine $w_j(t)$ and $A_j(t)$.

However, t_{0j} is not known in advance. We must choose t_{0j} so that $p_j(t)$ and $s_j(t)$ vanish simultaneously at time t_{1j}. It has been proved that, if $W_j(t)$ has a shape shown in Fig.2-1, then $p_j(t_{1j})$ is a monotone decreasing function of t_{0j}, and there exists a unique value of t_{0j} so that $p_j(t_{1j})$ is zero when $s_j(t)$ vanishes, provided that $f_2(w)$ is monotone increasing in w [1,8,11].

We have thus seen that if the $W_j(t)$ are known, the evaluation of the $A_j(t)$ is the same as for a single bottleneck. The complication in the general theory comes from the fact that the domain of attraction of the j th bottleneck may move as the queues evolve. Which bottleneck a commuter chooses will depend on the $p_j(t)$.

2.3. SPATIAL EQUILIBRIUM CONDITION

Suppose that for a certain work schedule time t, bottleneck costs $p_j(t)$'s are known at all bottlenecks. This means that if the three-dimensional demand space Ω is sliced horizontally at time t, we can evaluate the trip cost $TC_j(x,t) = f_1\{m_j(x)\} + p_j(t)$ for every point on this sliced

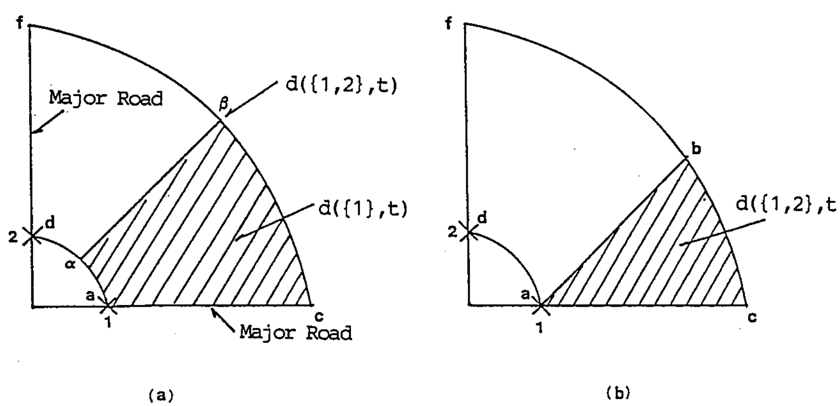

Fig. 2-3. Domain of Attractions

surface $\Omega(t)$, and identify which point is attracted to which bottleneck on the surface $\Omega(t)$.

Some points having a unique preferred bottleneck are called interior points. A point set $d(\{j\},t)$ is defined as the subset of $\Omega(t)$, which contains only points uniquely attracted to bottleneck j. Fig. 2-3 shows an example of partitions of $\Omega(t)$ for a geometry with a ring major road "ad" and two major radial roads leading to the working place in a quarter circular city. Finely spaced ring roads are provided around the working place as a center. If bottleneck costs $p_1(t)$ and $p_2(t)$ are nearly the same at time t, we expect a domain of attraction such as in Fig.2-3(a). Bottleneck 1 attracts points in the shaded area (interior region $d(\{1\},t)$) and bottleneck 2 attracts points in the unshaded area (interior region $d(\{2\},t)$).

On the other hand, some points have more than one equally good bottlenecks. These points, called boundary points, usually form lines. In Fig.2-3(a), bottlenecks 1 and 2 are equally preferred for points on line "$\alpha\beta$". This boundary line is similarly denoted as the set $d(\{1,2\},t)$. However, if $f_1(m)$ is a linear function, boundary points can also form a region due to a junction of major roads. In Fig.2-3(b), a junction of major roads "ac" and "ad" creates a boundary region $d(\{1,2\},t)$, when the travel cost between bottlenecks 1 and 2 is equal to $p_1(t) - p_2(t)$. The cheapest route between a home and the working place via different bottlenecks passes the common junction.

In general, at any moment t, the two-dimensional sliced demand surface can be partitioned into either interior or boundary points. This partition will change with time if differences in costs $p_j(t)$ for different j vary temporally.

If the partitions of $\Omega(t)$ do not contain any boundary regions, and there is a rate density $\rho(x,t)$ in the demand space Ω, the demand rate, $W'_j(t)$, attracted to bottleneck j can be represented as

$$W'_j(t) = \int_{|d(\{j\},t)|} \rho(x,t)\, dx . \tag{2.10}$$

Since the partitions $d(\{j\},t)$ are determined by the costs $p_j(t)$, $j=1,2,\ldots,J$, the $W'_j(t)$ are essentially functions of the $p_j(t)$ and the total demand $W'(t)$ which is a given function of t,

$$W'_j(t) = g_j(p_1(t), \ldots\ldots, p_J(t);t), \tag{2.11}$$

where the functions g_j depend on the demand space and network geometry. From the general relationship between curves $D_j(t)$ and $W_j(t)$, it follows also that

$$s'_j(t) = 1 - (1/\mu_j)g_j(p_1(t),\ldots,p_J(t);t). \tag{2.12}$$

This equation is the spatial equilibrium condition.

If some boundary regions exist, for example $d(\{i,j\},t)$, then $W'_j(t)$

would also include part of the demand from the boundary region. In Section 4.0, we will discuss how this demand must be split between two bottlenecks.

3.0. THE MORNING PEAK, WITH NO BOUNDARY REGIONS

For any particular network geometry and rate density $\rho(x,t)$, one could, in principle, determine the functions g_j in Eq.(2.12). The temporal equilibrium equations (2.6) and the spatial equilibrium equations (2.12) would then determine the time derivatives $p_j'(t)$ and $s_j'(t)$ as known functions of the $s_j(t)$ and $p_j(t)$ whenever $w_j(t) > 0$. Thus, we would have a system of ordinary differential equations for the $p_j(t)$ and $s_j(t)$. If, however, $w_j(t) = 0$, we know that $p_j(t) = s_j(t) = 0$.

If we knew when (or if) a queue first forms at each bottleneck, we could number the bottlenecks in order of the queue starting times

$$t_{01} \leq t_{02} \leq ,\ldots, \leq t_{0J}. \tag{3.1}$$

For $t \leq t_{01}$, no queue exists at any bottlenecks, so $p_j(t) = s_j(t) = 0$ for all j. For $t_{01} < t \leq t_{02}$, only $p_1(t)$ and $s_1(t)$ are non-zero, and they satisfy the equations

$$\begin{aligned} p_1'(t) &= f_3'\{s_1(t)\} , \\ s_1'(t) &= 1 - (1/\mu_1)g_1(p_1(t),0,0,\ldots,0;t). \end{aligned} \tag{3.2}$$

Since we know that $p_1(t_{01}) = s_1(t_{01}) = 0$, we can solve these equations (numerically) and determine $p_1(t)$ and $s_1(t)$ until time t_{02}.

For $t_{02} < t \leq t_{03}$, $p_j(t)$ and $s_j(t)$ are non-zero only for j = 1, 2, so we need integrate Eq.'s (2.6) and (2.12) only for j = 1, 2 with known values $p_1(t_{02})$, $s_1(t_{02})$, $p_2(t_{02})$, and $s_2(t_{02})$. In the next time interval $t_{03} < t \leq t_{04}$, we must solve the equations for j = 1, 2, 3, etc. Thus, for given values of the t_{0j}, one could, in principle, evaluate the $p_j(t)$ and $s_j(t)$ for all t.

As in the single bottleneck case, the t_{0j} must finally be determined so that when $s_j(t)$ vanishes at time t_{1j}, $p_j(t_{1j})$ also vanishes (for each j). We presume that a solution of these equations exists, but do not give a proof. If there were several bottlenecks, we expect that it would be computationally quite difficult to determine the t_{0j} so as to satisfy these conditions.

3.1. EXAMPLES — QUEUEING PATTERNS FOR J = 2

In order to illustrate some of the qualitative features of this model, particularly how the boundary motion affects the queueing pattern, we consider some special cases with two bottlenecks (J=2) and some special forms for the $f_3(s)$ and g_j in Eq.'s (2.6) and (2.12).

Suppose that there are two major roads 1 and 2 leading to a working place in a single core city, and bottlenecks exist on each of the two roads. Finely spaced minor roads are provided (Fig.3-1). The work starting time distribution of commuters is assumed to be the same everywhere; i.e., the rate density of trips has a product form,

$$\rho(x,t) = W'(t)\ \rho^*(x)\ , \tag{3.3}$$

where $\rho^*(x)$ = probability density of trips at location x.

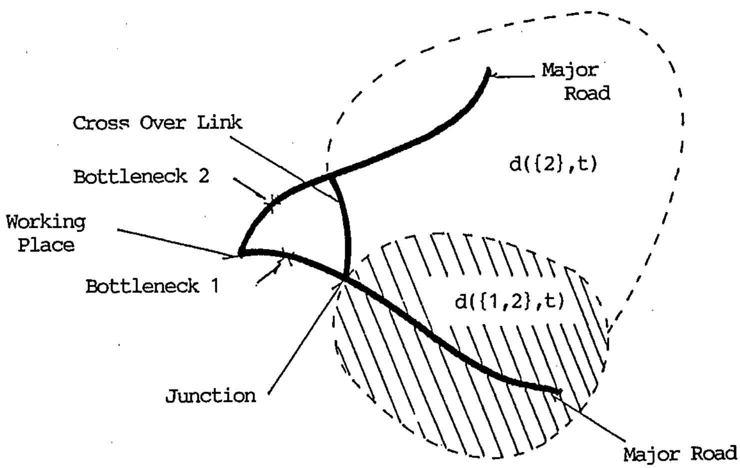

Fig. 3-1. A Two-Bottleneck Geometry.

The function $g_j(p_1(t),p_2(t);t)$ depends on the $p_j(t)$ only through the difference $p_1(t) - p_2(t)$. Furthermore, for many types of network geometries or for a sufficiently small $p_1(t) - p_2(t)$, we can approximate $g_j(p_1(t),p_2(t);t)$ as linear in $p_1(t) - p_2(t)$,

$$s_1'(t) = 1 - [W'(t)/\mu_1][\alpha - \beta\{p_1(t) - p_2(t)\}], \tag{3.4}$$

$$s_2'(t) = 1 - [W'(t)/\mu_2][(1-\alpha) + \beta\{p_1(t) - p_2(t)\}], \tag{3.5}$$

$$0 < \alpha < 1\ ,\ \text{and}\ \beta \geq 0,$$

for $w_1(t) > 0$ and $w_2(t) > 0$.

If no queue exists at either bottleneck, $p_1(t) = p_2(t) = 0$, a fraction α of trips is assigned to bottleneck 1, and $(1-\alpha)$ to bottleneck 2. We assume that $\alpha/\mu_1 > (1-\alpha)/\mu_2$: bottleneck 1 is more critical than bottleneck 2, and $f_3(s)$ is piecewise linear as in Eq.(2.8).

If $\beta = 0$, the two bottlenecks behave independently of each other. Each has a stationary domain of attraction and the queue behavior is as described as in Section 2.2. For $\beta > 0$, there are no queues until some time t_{01}, but there are several types of subsequent queueing patterns depending on queue starting times t_{0j}, β, and c_2.

If μ_2 is sufficiently large, no queue forms at bottleneck 2, $p_2(t) = 0$, for all t. After a queue forms at bottleneck 1, $p_1(t)$ can be evaluated from Eq.(2.9)

and
$$p_1(t) = c_1(t - t_{01}) \ , \qquad (3.6)$$
$$s_1'(t) = 1 - [W'(t)/\mu_1][\alpha - \beta c_1(t - t_{01})].$$

From this we see that the effect of $\beta > 0$ is to increase $s'_1(t)$, which means a reduction in $W_1(t)$. Thus, trips divert from bottleneck 1 to 2 as t increases.

If schedule delay $s_1(t)$ vanishes for the first time at some time t_{21} and $s_1'(t) < 0$ at $t = t_{21}$, then, $s_1(t)$ becomes negative for $t > t_{21}$. The $p_1'(t)$ switches from c_1 to $-c_2$ and remains at $-c_2$ until time t_{11} when $s_1(t)$ vanishes again, and presumably stays equal to zero thereafter. For $t_{21} < t < t_{11}$, $s_1'(t)$ will have the form

$$s_1'(t) = 1 - [W'(t)/\mu_1][\{\alpha - \beta p_1(t_{21})\} + \beta c_2(t - t_{21})]. \qquad (3.7)$$

If t_{01} were known, the right hand side of Eq.(3.6) would be known and the equation for $s_1'(t)$ could be integrated to give $s_1(t)$. By observing when $s_1(t) = 0$, we could determine t_{21} and $p_1(t_{21})$, so the right hand side of Eq.(3.7) can also be evaluated. We can continue to integrate $s_1'(t)$ to obtain $s_1(t)$. The t_{01}, however, must be chosen so that $p_1(t_{11}) = 0$ at the time t_{11} when $s_1(t_{11}) = 0$.

A hypothetical queueing pattern is illustrated in Fig.3-2. Figure 3-2 (b) shows $p_1(t)$ increasing linearly with t for $t_{01} < t < t_{21}$ and decreasing linearly for $t_{21} < t < t_{11}$, with slopes c_1 and $-c_2$ respectively. This causes the boundary to move as illustrated in Fig. 3-2 (c). The vertical height is the total area of a sliced demand surface $|\Omega(t)|$. It is divided into two partitions here: a partition for bottleneck 1, $|d(\{1\},t)|$; and for bottleneck 2, $|d(\{2\},t)|$. For $t < t_{01}$ the domain of attraction of bottleneck 1, $|d(\{1\},t)|$, stays constant, since $p_1(t) = p_2(t) = 0$. For $t_{01} < t \leq t_{21}$, $|d(\{1\},t)|$ decreases because $p_1(t)$ increases. For $t_{21} < t \leq t_{11}$, $|d(\{1\},t)|$ increases because $p_1(t)$ decreases; and it returns to the original size at time t_{11}. This motion of the boundary, in turn, determines the $W_1(t)$ in Fig. 3-2 (a) and queueing delay $w_1(t)$ and thus the curve $A_1(t)$ in Fig. 3-2 (a).

This is the type of queueing pattern that one would expect for sufficiently small value of βc_2 with a sufficiently large μ_2. If the boundary moves only slightly and sufficiently slowly, the pattern should be similar to that for $\beta = 0$ with each bottleneck serving a (nearly) specified demand.

If βc_2 is too large, however, one cannot construct a solution of the above type because there is a constraint on the rate at which the boundary can move after time t_{21}. In order for $s_1(t)$ to vanish for the second time at time t_{11}, given that $s_1(t) = 0$ and $s_1'(t) < 0$ at time t_{21}, it is necessary that $s_1'(t) > 0$ at time t_{11}. For sufficiently large βc_2, however, the last term of Eq.(3.7) will dominate and cause $s_1'(t)$ to be increasingly negative. Thus $s_1(t)$ will not return to zero at any time $t_{11} > t_{21}$.

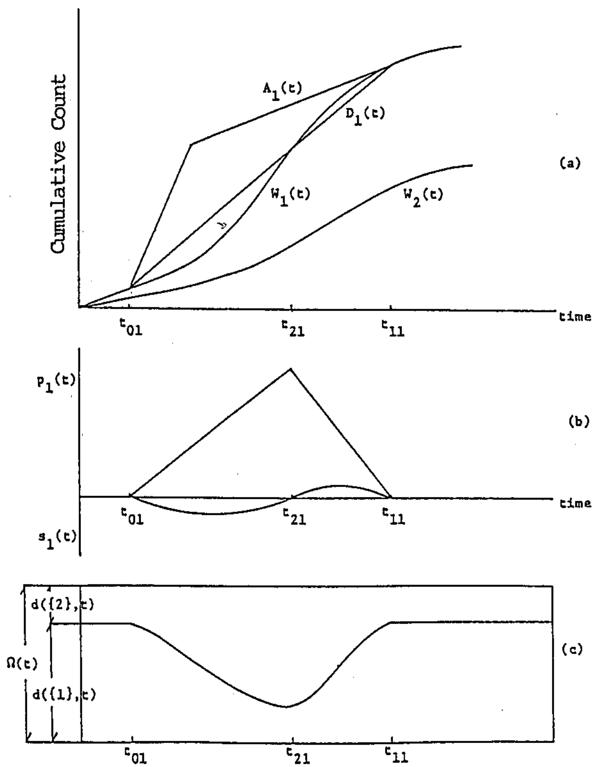

Fig.3-2. A Queueing Pattern for J=2, where Only Bottleneck 1 has a Queue--Pattern 1.

The only possible equilibrium in this case is for $s_1(t)$ and $s_1'(t)$ to vanish simultaneously at time t_{21}, and for $s_1(t)$ to stay equal to zero for a while as shown in Fig.3-3. The $p_1(t)$ for $t_{21} < t < t_{11}$ is now determined by Eq.(3.4),

$$s_1'(t) = 0 = 1 - [W'(t)/\mu_1][\alpha - \beta p_1(t)] ,$$

$$p_1(t) = \alpha/\beta - \mu_1/\{\beta W'(t)\}.$$

In this period, $p_1(t)$ is independent of t_{01}, t_{21}, and c_2, provided $p_1'(t) > -c_2$. The t_{01} and t_{21} must be determined so that $s_1(t)$ and $s_1'(t)$ both vanish at time t_{21}.

The time t_{11} when the queue vanishes is the time when $W_1'(t) = \mu_1$; that is, the time when the demand on bottleneck 1 at the original location of the boundary is equal to the maximum service rate of bottleneck 1. This time t_{11} can be known in advance from $W(t)$.

This evolution of the queueing pattern with a large βc_2 is quite different from that in the single bottleneck case. For a single bottleneck, there is no time period when the schedule delay stays equal to zero. This difference comes from the motion of the boundary. A queue cannot dissipate suddenly at time t_{21}, because this would cause the domain of attraction $d(\{1\},t)$ to increase suddenly, $W_1'(t)$ to exceed the maximum service rate μ_1, and $s_1(t)$ to become negative.

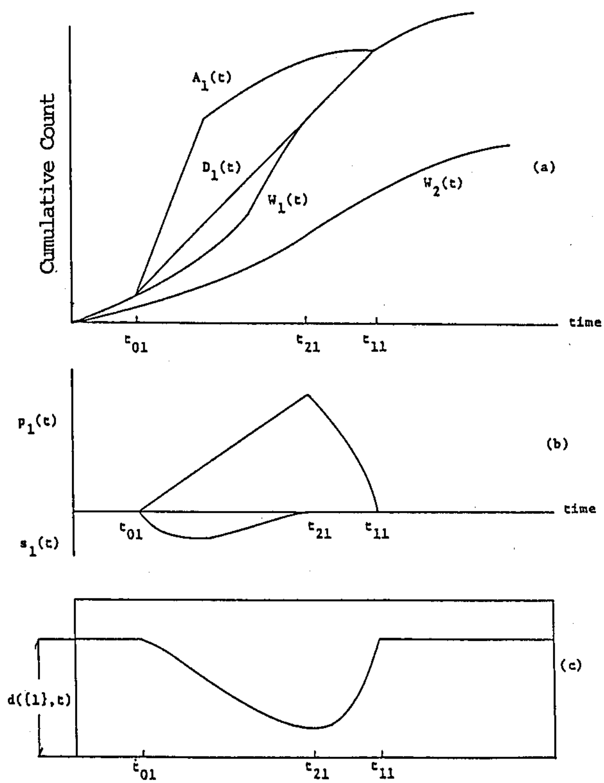

Fig.3-3. A Queueing Pattern for J=2, where Only Bottleneck 1 has a Queue--Pattern 2.

Figures 3-4 and 3-5 illustrate some typical queueing patterns if a queue forms at both bottlenecks 1 and 2. As was true when only one bottleneck has a queue, there will be a critical value of βc_2 at which the pattern changes. Figure 3-4 shows a pattern for βc_2 less than the critical value; Fig. 3-5 for βc_2 above the critical value. The details of how one can construct these and other patterns are described in more detail in reference [8].

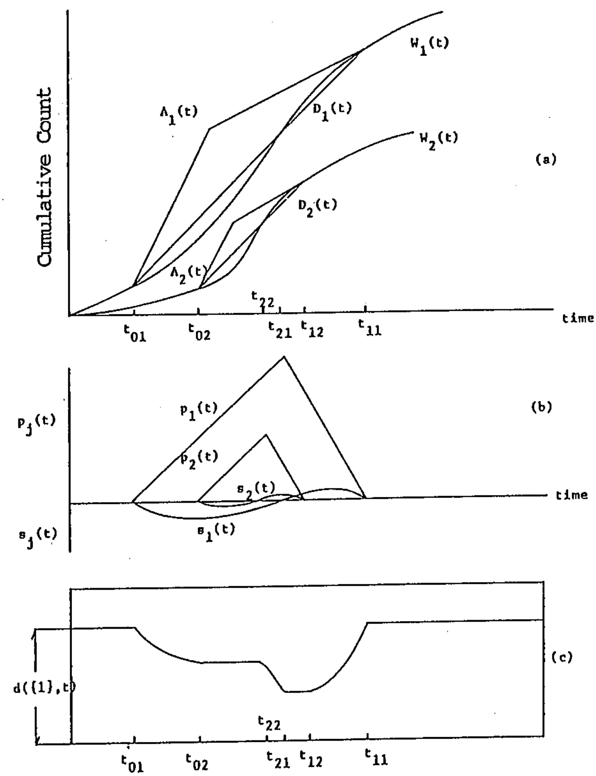

Fig.3-4. A Queueing Pattern for J=2, where Both Bottlenecks have Queues--Pattern 1.

If, in Fig.3-4, one know the times t_{01} and t_{02} when the queue first forms and the times t_{11} and t_{12} when they end, one could evaluate $p_1(t)$ and $p_2(t)$, which are piecewise linear curves with slopes c_1 or $-c_2$ as shown in Fig. 3-4 (b). This would then determine the motion of the boundary which depends on $p_1(t) - p_2(t)$. When $p_1(t)$ and $p_2(t)$ are both increasing or decreasing at rates c_1 or $-c_2$, the boundary is stationary as illustrated in Fig. 3-4 for the times $t_{02} < t < t_{22}$ and $t_{21} < t < t_{12}$. One obtains different patterns, however, depending on the relative ordering of the times t_{22}, t_{21} and t_{12}.

In Fig. 3-5, a large value of βc_2 forces $s_2(t)$ and $s_2'(t)$ to vanish simultaneously at times t_{22} and t_{32} and to remain zero between these times. Also $s_1(t)$ and $s_1'(t)$ must vanish simultaneously at some time t_{31} and remain zero thereafter. There are well-defined procedures for evaluating the various times $t_{01},\ldots\ldots,t_{11}$.

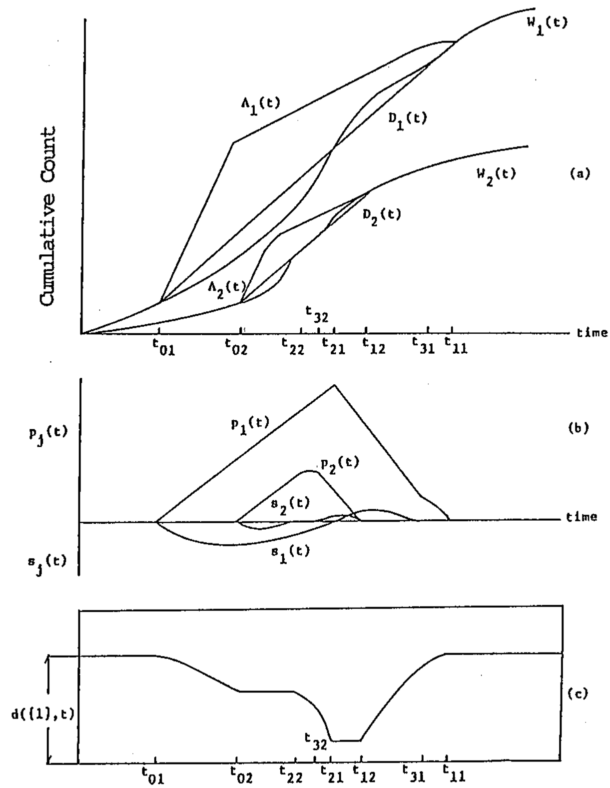

Fig.3-5. A Queueing Pattern for J=2, where Both Bottlenecks have Queues--Pattern 2.

4.0. THE MORNING PEAK, WITH BOUNDARY REGIONS

It is quite likely that some road network containing two or more major roads leading to the city center would also have roads over which traffic could be diverted from one bottleneck to another. Suppose, for example, that there was a cross over link between two major roads as illustrated in Fig. 3-1.

If $f_1(m)$ is linear, the travel cost of a commuter who normally would pass the junction with the cross over link on his way to bottleneck 1 is the sum of the travel cost from his home to the junction and the travel cost from the junction to the work place. If the travel cost from the junction to the work place via the cross over link and bottleneck 2 is larger than via bottleneck 1 by an amount C, such a commuter would actually choose bottleneck 1 only if $p_1(t) - p_2(t) < C$, independent of his home location. If , however, $p_1(t) - p_2(t) = C$, all commuters who pass the junction would find the cost equal via bottlenecks 1 and 2, thus there would be a boundary region $d(\{1,2\},t)$ as illustrated in Fig. 3-1.

If a queue forms first at bottleneck 1 at some time t_{01}, $p_1(t) - p_2(t) = p_1(t)$ will increase at least until time t_{02} when a queue forms also at bottleneck 2. If $p_1(t) - p_2(t)$ should reach the value C the boundary region will form and we expect that it will persist for some time as commuters shift so as to maintain this condition.

There is a slight complication if $f_3(s)$ is piecewise linear as in Eq.(2.8) because, with this $f_3(s)$, $p_1(t) - p_2(t)$ will remain constant after a queue forms at bottleneck 2 (whether or not there is a boundary region). If, however, $f_3(s)$ is strictly convex, $p_1(t) - p_2(t)$ will continue to increase even after time t_{02} until it has the value C. If $p_1(t) - p_2(t)$ remains equal to C, then $p_1'(t) = p_2'(t)$. It is still necessary, however, that the temporal equilibrium condition Eq.(2.6) be satisfied at both bottlenecks. Thus it is necessary that

$$f_3'\{s_1(t)\} = f_3'\{s_2(t)\}, \qquad (4.1)$$

while there is a boundary region.

If $f_3(s)$ is convex and $f_3'(s)$ monotone increasing, $f_3'(s)$ has a unique inverse [5] and consequently

$$s_1(t) = s_2(t) \ . \qquad (4.2)$$

This condition may seem rather surprising since the $s_1(t)$ is certainly increasing for some time after t_{01} while $s_2(t) = 0$. So, this condition cannot be satisfied until after a queue forms at bottleneck 2. Furthermore, the t_{01} and t_{02} must be chosen in such a way that by the time $p_1(t) - p_2(t)$ reaches the value C, $s_2(t)$ has increased to the value $s_1(t)$.

While the boundary region exists, Eq.(4.2) implies that the demand must split between the two bottlenecks so that $W'(t)$ is divided proportional to the μ_j, i.e.

$$W_j'(t) = W'(t)\mu_j/(\mu_1 + \mu_2), \qquad j = 1, 2. \qquad (4.3)$$

The integration of the equations for $p_j(t)$ and $s_j(t)$ is quite straightforward if a boundary region forms. As in the previous cases, the main problem is to determine times t_{01}, t_{02} etc when queues form or pattern switches. Some examples are given in reference [8].

5.0. CONCLUSION

This research develops methods to predict time-dependent traffic flow during the morning peak when queues on freeways cause major delays.

In general, time-dependent network analysis becomes very complicated if we simply extend current static network analysis methods to time-dependent situations. To reduce the analysis to a manageable size, we proposed several assumptions such as a flow independent travel speed, a many-to-one OD table, a common form of trip cost function, a point queue, only one bottleneck traversed by each trip, and so on. Particularly, the assumption of flow independent travel speed allows us to focus on only bottleneck sites. As long as freeways are major facilities for commuting, this assumption is reasonable.

Based on these assumptions, we propose a method to obtain the cumulative arrival curves at more than one bottleneck, given commuters' home locations and work schedules. This method is, in principle, straightforward: the problem reduces to solving first order differential equations for schedule delays and bottleneck costs (costs due to queueing and schedule delays) simultaneously. These differential equations cannot be solve analytically in general; however, they can be solved numerically.

The assumptions made here limit the applicability of the results, but it is possible to relax some of these assumptions for more realistic models. Particularly, it seems to be useful future works to study a case where the trip cost function is no longer common [10], and a tandem bottleneck case where commuters may enter bottlenecks more than once. Extensions of the method discussed here to more realistic models do not appear to be prohibitively difficult.

FOOTNOTES

1) If $f_3(s)$ is not continuously differentiable at s, particularly at s = 0:

$$f_3'(0-) \leq p'(t_w) \leq f_3'(0+).$$

2) It can be shown that if $f_3(s)$ is convex in s, there is a unique arrival curve A(t); and if $f_3(s)$ is strictly convex in s, not only the arrival curve but the assignment are unique. Even if the convexity is not strict, FIFW is

one of the equilibrium assignments.

3) Because the travel time between one particular bottleneck and the point working place is the same for everyone who passes the bottleneck, we can order a commuter's travel pattern as follows without changing his trip cost. Everyone spends the whole travel cost first, then joins the back of a queue, waits in the queue, and finally spends the schedule delay at his office.

4) The notation t is used from now on instead of t_w.

5) If $f_3(s)$ is piecewise linear in s as in Eq.(2.8), Eq.(4.2) is not necessary in order to satisfy Eq.(4.1). However, the spatial assignment satisfying Eq.(4.2) is one of the equilibrium spatial assignments.

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