Estimation of Time Dependent OD Matrices From Traffic Counts

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ABSTRACT

We propose the model for estimating time-dependent OD matrices from traffic counts in a general network with route choice activities. The model consists of two parts : (1) construction of the relationship between the time-dependent OD volumes and traffic counts at links and (2) estimation of a unique time-dependent OD matrices. In the first part, we define a three-dimensional network to relate OD matrices to traffic flow on links. We then propose a method of estimating route choice probabilities. In the second part, we employ the Entropy Maximizing method for the static OD matrices estimation and extend it to time-dependent model. As an extension, we propose a simplified method of estimating route choice probability and a method to utilize aggregated prior OD information. At the last, we apply the model to a test network and examine its validity.

INTRODUCTION

A model for estimating time-dependent OD(Origin-Destination) matrices is required to realize the optimum traffic control and planning. Many dynamic traffic simulation models have been developed in order to reproduce traffic conditions and evaluate policies of traffic control, signal control, one-way traffic control and so on. Such a dynamic model needs a time-dependent OD, especially one composed of small OD zones. However, it is hard to estimate OD flows directly from OD survey. In this study, we thus propose a dynamic estimator using time-dependent traffic counts to obtain time-dependent OD volumes. There has been some models estimating or updating time-dependent OD matrices from traffic counts. These models can be divided into two types, the Intersection models and the Network models. The Network model is more complicated than the former because of the consideration of route choice behavior. Nguyen *et al.*[1)] proposed the model on a general transit network. Yang *et al.*[2)] estimated OD matrices so as to minimize the integrated squared error between observed and predicted link traffic flows with an efficient solution method developed based on Fourier transformation. However, these models have not included time-dependent driver's route choice behavior. In the model developed by Cascetta *et al.*[3)], although they consider the driver's route choice behavior, as enumeration of all routes is required to estimate route choice probabilities, calculation does not seem to be applicable to a large network. Ashok and Ben-Akiva[4)] have also proposed the on-line estimator using the Kalman filtering. However, a ratio of traffic flow on each route and prior OD flows are required in this estimator.

TIME-DEPENDENT OD MATRICES ESTIMATION MODEL

Outline of the Model

This study develops the model to estimate time-dependent OD volumes in a general network, which consists of links and nodes. In this model, each link has time dependent link travel time which is however flow-independent and must be input in advance based upon the field observation. In the first part of the model, the relationship between OD flow and link flow is established by introducing a route choice probability determined from the time dependent link travel times. In the second part, a time-dependent OD matrix is uniquely estimated by applying the entropy maximizing method under constraints of the relationship between OD and observed link flows obtained in the first part.

Definition of Time Axis and Formulation

A vehicle trajectory on path k is drawn in the time-space graph as in Fig.1, in which time axis is divided into discrete time-intervals of equal length Δt, and time-interval h is defined as time interval $[h\Delta t\ ,\ (h+1)\Delta t]$. $T_a(h)$, travel time at link a at time-interval h, is assumed to be a multiple of an integer Δt. Hence, Δt must be sufficiently small so that change of link travel time over time can be well described. It is also assumed that $T_a(h)$, $v_a(h)$, link flow at link a at time-interval h and $q_w(h)$, OD flow departing from origin node of OD pair w at time-interval h, do not vary during each time-interval, which means that they stay constant values at the start of time-interval.

In Fig.1, a vehicle departing from an origin at time-interval h_r passes through several links along the path k and enters link a at time-interval $\tau_{ak}^{w}(h_r)$, which is arrival time at link a when a traffic on path k of OD pair w generates from a origin at the time h_r. Here, since $T_a(h)$ is predetermined, arrival time at link a, $\tau_{ak}^{w}(h_r)$, can be calculated by summing link travel times along the trajectory.

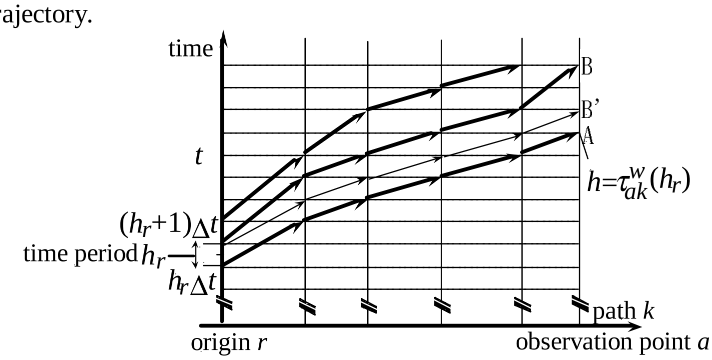


Figure 1 Time-Space Graph
(Descrete Time Axis)

If vehicles A and B actually travel as shown in Fig.1, we normally expect that trajectories of vehicles departing origin r during time interval h_r are spread between trajectories A and B. However, we have to note that, under the assumption above, those trajectories stay between A and B' because of the constant travel time in any time period of Δt.

Using a network with the discrete time axis, a relationship between OD flow and link flow can be written as:

$$v_a(h)\cdot\Delta t=\sum_w\sum_k p_{kw}\left(\tau_{ak}^{w\ -1}(h)\right)\cdot q_w\left(\tau_{ak}^{w\ -1}(h)\right)\cdot\delta_{ak}^{w}\cdot\Delta t\ , \tag{1}$$

where

$p_k(h)$:probability that traffic flow of OD pair w departing from its origin during time-interval h uses path k,

$\tau_{ak}^{w\ -1}(h)$:departure time-interval from an origin when a traffic flow on path k of OD pair w enters link a during time-interval h,

δ_{ak}^{w} : 1 ; if path k of OD pair w passes through link a,
0 ; *otherwise*.

This relationship can be also written as:

$$v_a(h)\cdot\Delta t=\sum_w\sum_{h_r} p_{aw}(h_r,h)\cdot q_w(h_r)\cdot\Delta t\ , \tag{2}$$

in which $p_{aw}(h_r\ ,\ h)$ means probability that a vehicle departing from origin node r of OD pair w during time-interval h_r enters link a during time-interval h. The $p_{aw}(h_r\ ,\ h)$ hence satisfies following:

$$\sum_h p_{aw}(h_r,h)=1\ .$$

Estimation of Route Choice Probability

Let us assume user's route choice probability such that

$$p_{kw}(h_r)=\mathrm{Prob}\left[C_{kw}(h_r)+\varepsilon_{kw}(h_r)\le C_{mw}(h_r)+\varepsilon_{mw}(h_r)\right],\ \forall m\ ,$$

where

$C_{kw}(h_r)$: cost of path k of a vehicle departing from origin r of OD pair w at time-interval h_r,

$\varepsilon_{kw}(h_r)$: an error term of $C_{kw}(h_r)$.

If error term $\varepsilon_{kw}(h_r)$ has the Wible distribution, we obtain the following well known Logit model :

$$p_{kw}(h_r) = \frac{\exp(-\theta C_{kw}(h_r))}{\sum_m \exp(-\theta C_{mw}(h_r))}, \tag{3}$$

where θ is the Logit parameter, and it must be given externally based on the route choice activity. If path cost $C_{kw}(h_r)$ is assumed a linear function of link costs along path k, $C_{kw}(h_r)$ is written as:

$$C_{kw}(h_r) = \sum_a C_a(\tau_{ak}^w(h_r)) \cdot \delta_{ak}^w, \tag{4}$$

where $C_a(h)$ is the cost of link a at time-interval h.

We here consider only a case in which the path cost is a linear function of the link costs. Since link travel time $T_a(h)$ is considered one of the most representative factor, which is assumed given as mentioned earlier, the link cost is also assumed to be given for all links at all time periods.

The $p_{kw}(h_r)$ is obtained from the optimization problem (P1) as shown below by extending the Fisk model on the static assignment to the dynamic model.

PROBLEM P1:

$$\min\left[\frac{1}{\theta}\sum_{w,k,h_r} p_{kw}(h_r)\cdot\log p_{kw}(h_r) + \sum_{a,h} C_a(h)\cdot p_a(h)\right],$$

s.t.

$$p_a(h) = \sum_{w,k} p_{kw}\left(\tau_{ak}^{w\,-1}(h)\right)\cdot\delta_{ak}^w,$$

$$\sum_k p_{kw}(h_r) = 1,\quad p_a(h)\geq 0,\quad p_{kw}(h_r)\geq 0,$$

in which $p_a(h)$ is the summation of probabilities entering link a at time-interval h for all OD pairs. We can prove that the solution of problem P1 is equivalent to the Logit type path choice probabilities shown in (3) (see Appendix).

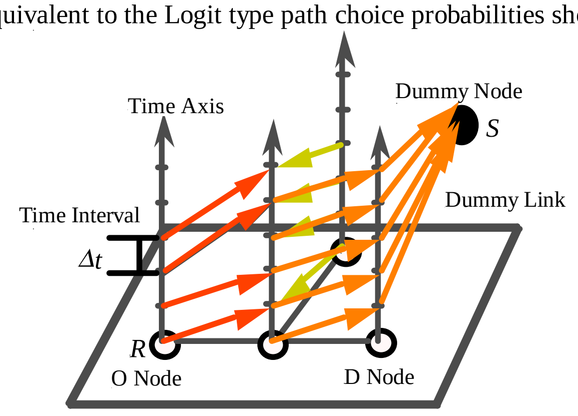


Figure 2 A Time-Space Network

To solve P1, let us introduce the three-dimensional time-space network, in which the one-dimensional time axis is added to the two-dimensional spatial network, as shown in Fig. 2. On this network configuration, actual link a is decomposed into several links depending on entering time periods. Since the vertical height of the link increases by its link travel time, slope of the link represents the velocity. And the FIFO can be known as a condition that links on the three-dimensional time-space network do not cross each other. In the case, a vehicle entering link a during time-interval h cannot be overtaken by a vehicle entering link a during time-interval $h+\Delta h$. So, link travel time $T_a(h)$ needs to be given so as to satisfy:

$$\frac{T_a(h+\Delta h)-T_a(h)}{\Delta h} \geq -1.$$

However, to adjust the three-dimensional network to the usual two-dimensional network, a new destination node S must be added as in Fig.2.

To consider the problem P1 on the three-dimensional network, variables in P1 have to be converted as follows. Origin node R on the three-dimensional network corresponds to a combination of origin node r and departure time period h_r on the two-dimensional network, and link A on the three-dimensional network corresponds to a combination of link a and entering time period h on the two-dimensional network. Thus, we can convert the problem P1 to the problem on the three-dimensional network by converting variables such as $p_{kw}(h_r)$ to p_{KW}, $C_a(h)$ to C_A, where p_{KW} is a probability that a vehicle having OD pair W(origin R, destination S) uses path K, and C_A is a cost of link A on the three-dimensional network.

The problem on the three-dimensional network converted from the problem P1 mentioned above can be solved in the same way as P1, and the result is written as:

$$p_{KW} = \frac{\exp(-\theta C_{KW})}{\sum_M \exp(-\theta C_{MW})}. \tag{5}$$

This means that the flow-independent Dial's assignment can be applied to solve the problem on the three-dimensional network.

Specifically, we can estimate p_{AW} by applying the Dial's assignment in which OD flow sets to 1 on any OD pair W on the three-dimensional network. Then, p_{AW} estimated above is equal to $p_{aw}(h_r, h)$ on the two-dimensional network. Because origin R on the three-dimensional network corresponds to origin r and departure time h_r on the two-dimensional network, destination S to destination s, and link A to link a and entering time period h, respectively.

Estimation of OD Matrices

As discussed in the previous section, a relationship between OD flow and link flow is formulated as (3) using the route choice probability. Suppose link flow is observed as $\tilde{v}_a(h)$ which consists of real link flow $v_a(h)$ and its error term $\varepsilon_a(h)$:

$$\tilde{v}_a(h) = v_a(h) + \varepsilon_a(h) = \sum_{w,h_r} p_{aw}(h_r, h) \cdot q_w(h_r) + \varepsilon_a(h). \tag{6}$$

Unknowns $q_w(h_r)$ must be estimated so that (6) is satisfied. The number of conditions (6) is the number of observed link (a) x the number of observed time-intervals (h), which is normally less than the number of unknowns, the number of OD pairs (w) x the number of time-intervals (h_r). Hence, we can find many sets of OD matrices which satisfy (6) and the problem is how we should choose a unique matrix among the candidate matrices satisfying (6).

In this study, we apply the Entropy Maximization method[5] to this time-dependent model in order to choose a unique matrix. Here, a prior OD flow departing from the origin r of OD pair w at time-interval h_r is denoted as $\tilde{q}_w(h_r)$. Then, OD flow $\bar{q}_w(h_r)$ and link flow $\bar{v}_a(h)$ can be estimated as:

$$\bar{q}_w(h_r) = \tilde{q}_w(h_r) \prod_{a,h} X_a(h)^{p_{aw}(h_r,h)} \tag{7}$$

$$\bar{v}_a(h) = \tilde{v}_a(h) X_a(h)^{-\frac{1}{\gamma}} \tag{8}$$

in which $X_a(h)$ is a parameter that can be obtained by solving

$$\tilde{v}_a(h) X_a(h)^{-\frac{1}{\gamma}} = \sum_{w,h_r} \left\{ p_{aw}(h_r, h) \tilde{q}_w(h_r) \prod_{a,h} X_a(h)^{p_{aw}(h_r,h)} \right\}, \forall a, h \tag{9}$$

The detailed derivation of (7), (8) and (9) is shown in reference[6]. Here, a parameter γ means a weight of link observed errors. The larger γ is, the lower observed errors are evaluated, and if $\gamma = \infty$, it is the same as normal entropy maximization method in which no errors are considered.

SOME EXTENSIONS ON THE MODEL

Aggregation of Time Axis and Simplified Estimation of Route Choice Probabilities

The model proposed in previous section needs to set time interval Δt so short as to describe changes of link cost (e.g. Δt will be less than about 30 seconds.). On the other hand, we usually need to estimate OD matrices of longer time interval , such as 15-60 minutes. Moreover, OD matrices at every Δt time interval will not preferable because stochastic changes are more dominant than trend changes. Thus, it is practical to set much longer time interval than Δt for estimation of OD matrices.

In this section, we discuss the simplification of estimating link use probability for longer time interval. We re-formulate a relationship between an OD flow and link flow for longer time interval assuming that OD flow rate does not change through the long time interval $\Delta T = m \cdot \Delta t$, where m is positive integer. For long time interval ΔT, we define aggregated time-interval $H(i)$ as follows:

$H(i)$ = a set of $\underline{m}$ short time-intervals Δt included in the time section $[(i\text{-}1)\Delta T, i\Delta T]$, $i = 1,2,...,H/m$.

If $Q_w(i)$ is OD flow rate for OD pair w at the i-th long time-interval ΔT , (2) becomes

$$v_a(h) \cdot \Delta t = \sum_w \sum_i Q_w(i) \sum_{h_r \in H(i)} p_{aw}(h_r, h) \cdot \Delta t \cdot$$

Furthermore, let us aggregate link flow through the j-th long time-interval ΔT . So

$$\sum_{h \in H(j)} v_a(h) \cdot \Delta t = \sum_w \sum_i Q_w(i) \sum_{h \in H(j)} \sum_{h_r \in H(i)} p_{aw}(h_r, h) \cdot \Delta t \,. \tag{10}$$

If we define $V_a(j)$ and $P_{aw}(i , j)$ as shown below, the above is simply rewritten as:

$$V_a(j) \cdot \Delta T = \sum_{w,i} \{ P_{aw}(i, j) \cdot Q_w(i) \cdot \Delta T \}, \tag{11}$$

where

$$V_a(j) = \sum_{h \in H(j)} v_a(h) \cdot \frac{\Delta t}{\Delta T} = \frac{1}{m} \sum_{h \in H(j)} v_a(h), \tag{12}$$

$$P_{aw}(i, j) = \frac{1}{m} \sum_{h_r \in H(i)} \sum_{h \in H(j)} p_{aw}(h_r, h) \,. \tag{13}$$

$P_{aw}(i , j)$ is a probability that a vehicle, departing from O node of OD pair w at the i-th time-interval ΔT , will be observed in the link a at the j-th time-interval ΔT. Since (2) and (11) have the exactly same form, OD matrices for longer time interval ΔT can be estimated by the same method as for small time interval Δt.

Next we consider efficient estimation of $P_{aw}(i , j)$ for longer time interval ΔT. Basically, $P_{aw}(i , j)$ at the i-th and j-th aggregated time-interval ΔT can be estimated from (13) if probability $p_{aw}(h_r , h)$ are obtained at all h_r and h by the method proposed in previous section. However, the estimation of $p_{aw}(h_r , h)$ for every h_r and h using small time interval Δt is quite tedious. To decrease calculation times, we make some assumption that route choice activities or link use probability does not change during a longer time period of $\Delta\tau = c\Delta t$. Here c is a positive integer, which is the number of short time-intervals Δt included in $\Delta\tau$. We assume $\Delta\tau = c\Delta t \le m\Delta t = \Delta T$ and m/c is integer. Practically speaking, ΔT and $\Delta\tau$ might be 60 minutes and 30 minutes or so. The n-th time-interval of $\Delta\tau$ means $[\{(n\text{-}1)c+1\}\Delta t, nc\Delta t]$ which is denoted as $N(n)$, and the initial time of $N(n)$ is written as $s(n)=(n\text{-}1)c+1$

Now we assume that route choice probabilities do not change at each time-interval of $\Delta\tau$ and vehicles departing from the same origin during the same time-interval $\Delta\tau$ have the same route choice probabilities. That is,

$$p_{aw}(s(n) + l, h) = p_{aw}(s(n), h - l)\text{, } l = 0, 1, 2, ..., c - 1.$$

Thus, $p_{aw}(h_r , h)$ can be written as:

$$p_{aw}(h_r, h) = p_{aw}(s(n) + l, h) = p_{aw}(s(n), h - l),\ l = h_r - s(n),\ h_r \in N(n),\ \forall a, w, n, h \tag{14}$$

From (12), $p_{aw}(h_r , h)$ for all h_r and h can be estimated by calculating only probabilities at the first short time-interval Δt of any the n-th time-interval $\Delta\tau$. Then, aggregated time link use probability $P_{aw}(i , j)$ can be estimated from (13).

Use of Spatially or Temporally Aggregated Prior OD Flow

The OD estimation model proposed in the study needs prior OD matrices $\bar{q}_w(h_r)$ for the target OD. However, in many cases, time-dependent prior OD matrices for fine zone-to-zone level are not available, but more global (aggregated) matrices such as daily OD matrices for larger zones. In this section, we discuss how spatially aggregated prior OD information can be utilized.

First of all, spatially and/or temporally aggregated OD can be expressed as linear summation of the minimum unit of OD flow, $q_w(h_r)$. Aggregated OD flow is denoted as g_u , $u \in U$, in which u is number of aggregated OD flow, and U is a set of aggregated OD flow. Then, aggregated OD flow can be formulated by using minimum OD flow as

$$g_u = \sum_{w,h_r} a_u(w,h_r) \cdot q_w(h_r), \tag{15}$$

where

$\phi(u)$: a set of OD pair and departure time (w,h_r) included in aggregated OD flow g_u , $u \in U$,

$a_u(w,h_r)$ $q_w(h_r)$: a parameter that expresses contribution of $q_w(h_r)$ to g_u. $a_u(w,h_r)$ is 1 if $(w,h_r) \in \phi(u)$ and 0 otherwise.

Now we use $\bar{g}_u$ for prior aggregated OD flow, and η_u for deviation of $\bar{g}_u$ from g_u:

$$\begin{aligned} \bar{g}_u &= g_u + \eta_u \\ &= \sum_{w,h_r} a_u(w,h_r) q_w(h_r) + \eta_u \end{aligned} \tag{16}$$

The (16) is very similar to (6) : $u \in U$ corresponds to $(a,h) \in W$, g_u corresponds to $a_u(w,h_r)$ and $a_u(w,h_r)$ corresponds to $p_{aw}(h_r,h)$. Therefore, by substituting $a_u(w,h_r)$ for $p_{aw}(h_r,h)$ and $\bar{g}_u$ for $a_u(w,h_r)$ in (7),(8),(9), the following equation can be obtained.

$$g_u \cdot X_u^{-\frac{1}{\gamma}} = \sum_{w,h_r} \left\{ a_u(w,h_r) \bar{q}_w(h_r) \prod_u X_u^{a_u(w,h_r)} \right\}, \forall u$$

From this equation, X_u can be solved, and $\bar{q}_w(h_r)$ can be obtained as shown below by substituting $X_a(h)$ for X_u in (7):

$$\bar{q}_w(h_r) = \bar{q}_w(h_r) \prod_{a,h} X_a(h)^{p_{aw}(h_r,h)} \cdot \bar{g}_u \prod_u X_u^{a_u(w,h_r)}.$$

NUMERICAL EXAMPLES

We apply the model discussed above to a test network as shown in Fig.3. The time interval used in the model is Δt=10 sec, $\Delta\tau$=30 min., and ΔT=1 hour and we estimate OD matrices for every 1 hour. The experiment is carried out in the following way. First, link travel time are given for all links at every time-interval Δt. Link use probabilities $P_{aw}(i\ ,j)$ for every ΔT=1 hour are estimated by the Dial's assignment on the three-dimensional network from (15) and (14). Here, the Dial's parameter is assumed θ=0.001[1/sec]. Next, real OD demand for every 1 hour is assigned to each link based on link use probabilities $P_{aw}(i\ ,j)$. These link use probabilities and link flows are used in the model as input data. All link flows assumed to be observed and hourly OD flow is used for the prior matrices of the model. Using these data, OD matrices were estimated by proposed OD estimation method using parameter γ = 1.0. We examine the model behavior by comparing OD estimates with real OD matrices assumed.

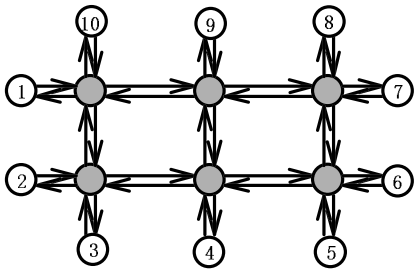


Figure 3 Test Network

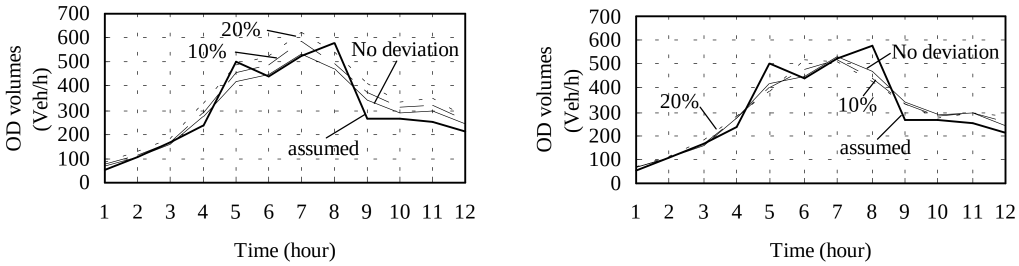
Table 1 Result of Simulation

	prior OD	observed error	correlation coefficient	RMSE (Veh/h)	PRMSE (%)
Case 1	A	A	0.456	191.6	80.5
Case 2	B	A	0.953	51.0	21.4
Case 3	B	B	0.950	52.0	21.8
Case 4	B	C	0.940	55.7	23.4
Case 5	C	A	0.947	52.8	22.1
Case 6	D	A	0.937	56.3	23.7
Case 7	E	A	0.925	59.9	25.2
Case 8	E	B	0.924	60.2	25.3
Case 9	E	C	0.916	63.1	26.5

prior OD A:no prior OD, B:no deviation,C:max 10%, D:max 20%, E:round off hundreds

observed error A:no error, B:max 10%, C:max 20%

We examine several cases on the different conditions of observed link flow and prior OD flow. Three patterns are considered on observed link flow: no error, maximum 10% error, and 20% error. In a similar way, four patterns are considered on the prior OD flow: no deviation, maximum 10% deviation, maximum 20% deviation, approximated prior OD rounded off to the hundred



(a)No Link Observed Error Cases (b) No Prior OD Deviation Cases

Figure 4 Time Variation of Estimated OD flow

digit. We examine nine cases in combination with these above cases.

The results of these experiments are shown in Table.1, which describes correlation coefficients between estimated OD matrices and assumed real OD, Root Mean Square Errors (RMSE) and Percent Root Mean Square Errors(PRMSE). First, let us see the correlation coefficient between OD estimates and real OD matrices. In the case that no prior OD flow is given (Case 1), the correlation coefficient is very low ; however, we can get rather good result in the cases with prior OD flows (Case2-9). which agrees with the same result as previous studies on static OD estimation. In the Entropy Maximizing Method, to get more precise OD matrices, it is a major factor to give more precise prior OD information close to real OD matrices.

Secondly, let us examine a effect of errors observed on link flows. The correlation coefficient get lower and RMSE(or PRMSE) increases when observed error gets larger(Case 2,3,4 and Case 7,8,9). However, in the case that observed error is maximum 10%(Case 3,8) the correlation coefficient, RMSE and PRMSE is almost same in comparison with the case that observed error does not exist(Case 2,7). Although we cannot say so definitely, it shows that a effect of observed errors of link flows on the accuracy of the model is not so large if observed error is sufficiently small.

Thirdly, consider effect of deviation of prior OD matrices. Four cases(Case 2,5,6,7) are tested in cases without observed link flow error. From these results, the larger deviation of prior OD matrices is, the worse accuracy of the model is. It shows that deviation in prior OD matrices have an effect on the accuracy of the model.

Fig.4 indicates hourly OD volumes from O node 6 to D node 1. This is the case that hourly OD volumes is the largest of all OD pairs in the network and its prior OD volumes have positive deviation from assumed ones. Fig.4(a) is the case where observed link flow error does not exist (Case 2,5,6), and (b) is the case where the deviation of prior OD matrices does not exist (Case 2,3,4). These figures indicate good fitness of time variation between estimated OD flows and assumed one. Note that the estimated OD flow varies uniformly in accordance with the deviation of prior OD matrices in the case of (a), but not uniformly in the case of (b).

CONCLUSION

In the study, we propose the model estimating time-dependent OD matrices from traffic counts in the general network. The model consists of two parts : (1) construction of the relationship between the time-dependent OD volumes and traffic counts at links and (2) estimation of the time-dependent OD. For the practical use, we propose the simplified method of estimating route choice probability and the method to use aggregated prior OD information.

We apply the model to a test network. The main results that have been made in this examples are as follows: (1)We can get rather preferable OD estimator in the case that prior OD matrices are given. (2) It is assumed that a effect of observed errors of link flows on the accuracy of the model is not so large if observed error is sufficiently small, but deviation of prior OD matrices have an effect on the accuracy of the model. (3) We can see good fitness of time variation between estimated OD flows and assumed one using the model.

We cannot mention the theoretical study on estimation errors or caused by link observed error, link use probabilities error and so on. So further study on such area must be made based on reliability analysis of the model.

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APPENDIX

The, Lagrangean of problem (P1), *L*, is:

$$L = \frac{1}{\theta}\sum_{w,k,h_r} p_{kw}(h_r)\cdot \log p_{kw}(h_r) + \sum_{a,h} C_a(h)\cdot p_a(h) + \sum_{w,h_r}\lambda_w(h_r)\left(\sum_k p_{kw}(h_r) - 1\right)$$
$$+\sum_{a,h}\mu_a(h)\cdot\left\{p_a(h) - \sum_{w,k} p_{kw}\left(\tau_{ak}^{w\;-1}(h)\right)\cdot\delta_{ak}^w\right\}.$$

Partial derivatives of *L* with respect to both unknown variables, $p_{kw}(h_r)$ and $p_{kw}(h_r)$, are

$$\frac{\partial L}{\partial p_{kw}(h_r)} = \frac{1}{\theta}\left\{\log p_{kw}(h_r) + 1\right\} + \lambda_w(h_r) - \sum_a \mu_a\left(\tau_{ak}^w(h_r)\right)\cdot\delta_{ak}^w$$

$$\frac{\partial L}{\partial p_a(h)} = C_a(h) + \mu_a(h)$$

Thus, by Kuhn-Tucker's condition,

$$\frac{1}{\theta}\left\{\log p_{kw}(h_r) + 1\right\} + \lambda_w(h_r) - \sum_a \mu_a\left(\tau_{ak}^w(h_r)\right)\cdot\delta_{ak}^w = 0\cdot$$

can be derived if $p_{kw}(h_r)$ is positive.

Thus, route choice probability of path *k* is written as:

$$p_{kw}(h_r) = \exp\left\{-\theta\cdot\sum_a C_a\left(\tau_{ak}^w(h_r)\right)\cdot\delta_{ak}^w\right\}\cdot\exp\left\{-\theta\cdot\lambda_w(h_r) - 1\right\}.$$

Using $\sum_k p_{kw}(h_r) = 1$, the above equation can be rewritten as:

$$p_{kw}(h_r) = \frac{\exp\left\{-\theta\cdot\sum_a C_a\left(\tau_{ak}^w(h_r)\right)\cdot\delta_{ak}^w\right\}}{\sum_m \exp\left\{-\theta\cdot\sum_a C_a\left(\tau_{am}^w(h_r)\right)\cdot\delta_{am}^w\right\}}.$$
$$= \frac{\exp\left\{-\theta\, C_{kw}(h_r)\right\}}{\sum_m \exp\left\{-\theta\, C_{mw}(h_r)\right\}}$$

This equation corresponds to the route choice probability defined in.(4). Thus, it is proved that the solution of problem (P1) gives the route choice probability defined in the former section.