A Theoretical Analysis on Departure Time Choice for Morning Commute Traffic Considering Individual Variation in Time Value and an Application to Road Pricing

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SUMMARY

This study theoretically analyzes users' departure time choice for morning commute traffic considering individual variations of time value and applies this analysis to a road pricing scheme. TDM policies have been proposed to mitigate social problems of traffic congestion. Although ITS technology is creating many instruments for TDM, we must have tools that evaluate such policies to acquire user acceptance. This study provides a theory for this evaluation. We construct a simple structure of user's behavior with numerical method. Each traveler is assumed to choose his/her departure time so as to minimize his/her cost, which consists of queuing delay on a bottleneck and schedule delay at the destination. We analyze this numerical problem graphically for easy applications to the congestion toll evaluation. It is concluded that the congestion toll which completely eliminates queuing delay exists even with individual variation of time values but the toll changes traveler's behavior and their costs.

INTRODUCTION

This study theoretically analyzes user's departure time choice for morning commute traffic considering individual variation of time value in order to evaluate traffic demand management policies, especially road pricing schemes.

Traffic congestion in morning commute is a serious problem for many large cities. Traffic demand management (TDM) policies have been proposed to mitigate the problem. ITS technology is creating many instruments for TDM, we, however, must have tools that evaluate policies and make people accept TDM policies besides ITS technologies. For example, although the electronic toll collecting system (ETC) is a major part of ITS and enables us to introduce an advanced toll system to mitigate congestion, the system may make congestion worse or bring major unfairness if it is not designed based on the evaluation study.

We now consider how to evaluate the policy that disperses travel demand over time. Traffic congestion may occur when many people want to use the same road at the same time. If we can lead some of them to choose different time, congestion may disappear even with the same amount of total demand. To analyze people’s decision of departure time choice, there have been some theoretical studies called "Departure time choice problem.[1][2][3]" These studies have handled commute traffic, which has strict time constraint on destinations: work starting time.

It is important to consider individual variation in time value when we consider a road pricing scheme, which is one of methods to disperse travel demand over time. Newell have solved[4] the departure time choice problem considering individual variation in time value at a single bottleneck, which is however, very complex. We rebuilt this theory to make an analysis clear and visual so as to be able to apply it to more advanced problems such as a road pricing scheme.

From this analysis, we learn how a travel cost and behavior may change under a road pricing scheme and that we can determine only one congestion toll even when there is individual variation in time value.

A MODEL WITHOUT CONGESTION TOLL

We made a mathematical model for analyzing decision of travelers. There is one bottleneck between a residential area and a working area (figure 1) whose capacity is constant and service is FIFO. All travelers must use this bottleneck.

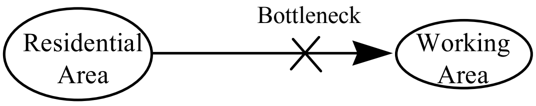

Figure 1. The network considered.

All travelers are commuters and have their time constraints of work starting times. Given work starting time of each traveler, we can know his/her desired departure time from the bottleneck, t_w, assuming travel time from the bottleneck to his/her working place does not change over time.

Decision of a traveler is described just with t_d , which is bottleneck departure time, and we assume that each traveler chooses t_d to minimize his/her traveling cost. No traveler is assumed to arrive his/her working place after work starting time. This means that t_d must not be larger than t_w. The travel cost is determined from queuing delay on bottleneck (w) and schedule delay (s), which is defined as waiting time for work on his/her work place, and simply written as t_w-t_d. The function w can be described using just one argument t_d because of the FIFO assumption. These w and s is shown as figure 2. The cost p is therefore defined as

$$p = c_w\, w(t_d) - c_s\, s\,(t_w, t_d) = c_w\, w(t_d) - c_s\,(t_w - t_d), \qquad (1)$$

where c_w is a cost per unit time of waiting time at bottleneck, and c_s is a cost per unit time of schedule delay. These parameters vary over individual commuters and the distributions of c_w and c_s values are assumed to be given.

We assumed that the system reaches equilibrium, which is defined as »A traveler cannot decrease his/her costs by changing departure time t_d." Our final goal is to determine $w(t_d)$ and to know t_d for all travelers. Based upon presumptions above, we analyzed the equilibrium pattern.

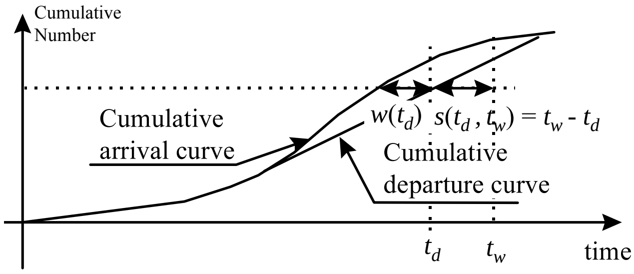

Figure 2. Definition of w and s on cumulative curves

First, we can describe the property of each traveler with t_w and c_w / c_s. If γ is defined as c_w / c_s, we can distribute all travelers on a 2-dimentional plain (t_w ,γ). The analysis described below is explained on this plain. Since each traveler tries to minimize his/her cost p, $dp/dt_d = 0$:

$$\frac{dp / c_w}{dt_d} = \frac{dw}{dt_d} - \gamma = 0 \quad \text{or} \quad t_d = t_w . \tag{2}$$

These equations consist of just γ and t_w. This is the reason that all travelers are characterized with (t_w, γ).

Second, we define a function called $\gamma^+(t_w)$, which divides all travelers into two groups: "early-arrival group" who arrive at their working place before work starting time ($t_d < t_w$) and "on-time-arrival group" who arrive at their working place just on work starting time ($t_d = t_w$). This function is single-valued, as shown in figure 3. If traveler's γ is smaller than $\gamma^+(t_w)$, the traveler is early-arrival. If traveler's γ is equal to or larger than $\gamma^+(t_w)$, the traveler is on-time-arrival. This can be explained as follows. Let us consider two travelers A and B with some t_w, but choose different departure time t_{dA} and t_{dB} because of the different time value ratio γ_A and γ_B. Based upon the assumption of the equilibrium, any travelers cannot decrease cost p by changing t_d, So,

$$\left\{w(t_{dA}) - \gamma_A(t_w - t_{dA})\right\} - \left\{w(t_{dB}) - \gamma_A(t_w - t_{dB})\right\} \geq 0 \qquad \text{and} \tag{3}$$

$$\left\{w(t_{dB}) - \gamma_B(t_w - t_{dB})\right\} - \left\{w(t_{dA}) - \gamma_B(t_w - t_{dA})\right\} \geq 0 \ . \tag{4}$$

Subtracting eq.(4) from eq.(3), we obtain

$$(\gamma_B - \gamma_A)(t_{dB} - t_{dA}) \geq 0 \tag{5}$$

Equation (5) means that t_{dB} is smaller than or equal to t_{dA} when γ_B is smaller than γ_A. So, when we set $\gamma^+(t_w)$ as the minimum value of   which travelers whose desired bottleneck departure time is t_w and t_d is equal to t_d have, a traveler with γ smaller than $\gamma^+(t_w)$ take $t_d < t_w$ ("early-arrivals") and otherwise, a traveler takes $t_d = t_w$ ("on-time-arrivals").

Third, we can calculate $w(t_d)$ and know t_d for each traveler just using this function $\gamma^+(t_w)$. Figure 3 shows how we can get $w(t_d)$ and t_d graphically. His/her bottleneck departure time t_d is determined as the time when $\gamma^+(t_w)$ is equal to his/her γ. Details are written below:

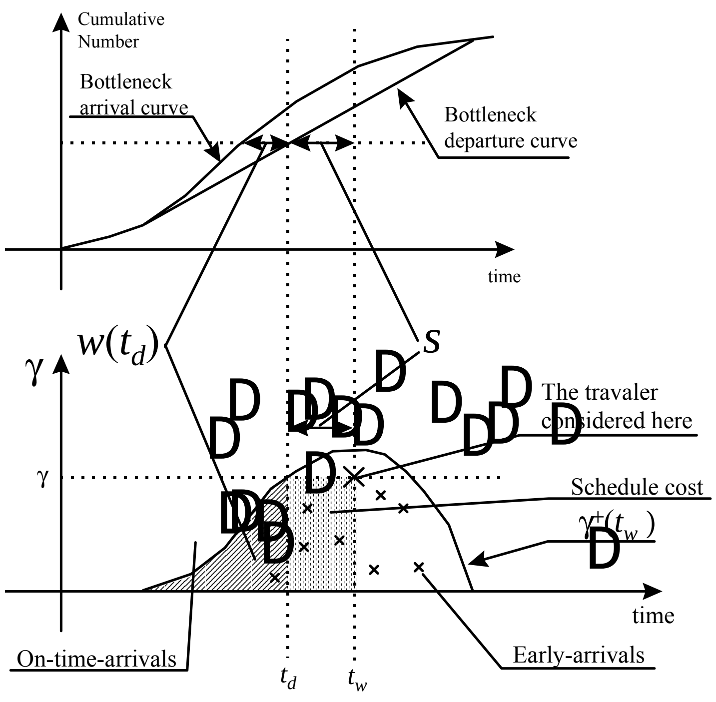

Figure 3. Graphical method for determinig t_d and $w(t_d)$.

$$w(t_w) = w(t_d) + \gamma^+(t_w)(t_w - t_d) \tag{7}$$

We assume one of on-time arrivals whose property is (t_w, $\cdot$ (t_w)) and one of early-arrivals whose property is (t_w- t_w, $\cdot$ (t_w)) (t_w>0) and his/her bottleneck departure time is t_d. Considering continuity of cost p, is established. Because on-time-arrivals has no t_d which is not equal to t_w, t_d must be t_w. Thus, early-arrivals whose is $^+$(t_w) depart from bottleneck on t_w.

If a traveler is one of early-arrivals, $w(t_d)$ is described with the hatched area in fig. 3. This is because from equation (2),

$$\frac{dw}{dt_d} = \gamma \tag{8}$$

This equation is only applicable to early-arrivals. Early-arrivals depart when γ^+ is increasing and a traveler with γ is departs from bottleneck at time t_d = $^{+-1}$( ) So,

$$w(t_d) = \int_{t_0}^{t_d} \frac{dw}{dt_d^*} dt_d^* = \int_{t_0}^{t_d} \gamma^+(t_d^*) dt_d^*, \tag{9}$$

where t_0 is congestion starting time, which should be during γ^+ is increasing.

The cost of early-arrivals p/c_w is equal to [hatched area]+[dotted area],

$$p / c_w = w(t_d) + \gamma(t_w - t_d) = \int_{t_0}^{t_d} \gamma^+(t_d^*) dt_d^* + \gamma(t_w - t_d) \quad . \tag{10}$$

Note that some early-arrivals with different departure time t_w have same departure time t_d if their γ is the same. This means that the schedule delay may not be the same even when each traveler has same t_d.

We have discussed queuing delay and schedule delay for early arrivals, so far. Let's now consider delay for on-time arrivals. Because of on-time arrival, a traveler can depart at his/her desired departure time t_w: $t_w = t_d$. If $\gamma^+(t_d)$ has a positive slope, the traveler should be delayed as the same amount as an early-arrival traveler departing at the same time $t_w = t_d$. On the other hand, if $\gamma^+(t_d)$ has a negative slope, since travel cost must be continue over early and on-time arrivals,

$$w(t_w) = w(t_d) + \gamma(t_w - t_d) \tag{11}$$

where t_d is departure time of early arrivals whose desired time t_w satisfies $\gamma = \gamma^+(t_w)$.

The departure flow on bottleneck on each time can be calculated from $\gamma^+(t_w)$. This is written as :

$$\begin{aligned} &(\text{flow}) = (\#\text{ of early - arrivals}) + (\#\text{ of on - time - arrivals}) \\ &(\#\text{ of early - arrivals}) = \left(\frac{d\gamma^+}{dt_d}\right)^{-1} \frac{dF}{dt_d}(W(t_d^*) - W(t_d)) \\ &(\#\text{ of on - time - arrivals}) = \frac{dW}{dt_d}\left(1 - F\left(\gamma^+(t_d)\right)\right), \end{aligned} \tag{12}$$

where $W(t_w)$ is the cumulative distribution of t_w and $F(\gamma)$ is the cumulative distribution of γ. The t_d* is defined as the time when travelers with the same switches their behavior from early arrival to on-time arrival (See fig.4). Note that the distribution of t_w is independent on the distribution of . We visualize this equation on fig.4. The flow rate from the bottleneck must be equal to the capacity μ with a queue. This condition and equation (12) determines $\gamma^+(t_w)$.

ESTIMATION OF TRAVELERS' PROPERTIES

We have explained the theory that calculate waiting time and travelers' behavior from travelers' properties (t_w,). We may have, however, no knowledge of their properties at all in many cases. In this section we show a method for estimating distribution of   from $w(t_d)$, which can be measured with traffic counters, and some assumption for distribution of t_w.

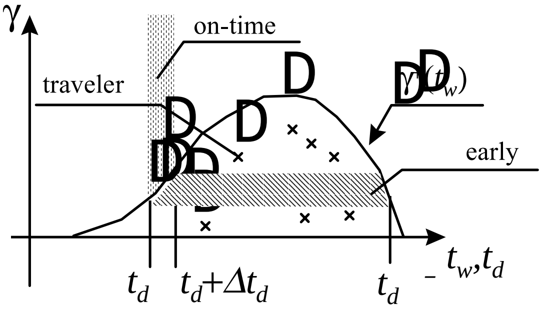

Figure 4. Determination of $\gamma^+(t_d)$. The number of travelers in dotted area (on-time-arrivals) and hatched area (early-arrivals) must be equal to $\Delta t_d \times$ [capacity of the road]

First, we get the function $^+$. We can calculate this with equations (9) and (11).
Second, we estimate $F(\ )$ from equation (12), which can be rewritten as

$$\mu = \frac{dW}{dt_d}(1 - F(\gamma^+(t_d))) \qquad (13)$$

when $\gamma^+(t_d)$ is decreasing. Note that we cannot know $F(\gamma)$ for $\gamma >$ max[γ^+] from this equation. This is because on-time-travelers whose γ is larger than max[γ^+] never change their behavior even if they change γ within any value [max[γ^+]..1], therefore $F(\gamma)$ for $\gamma >$ max[γ^+] affect no traveler's behavior. We must also point out that we can know F(γ) only with W(t_d) when $\gamma^+(t_d)$ is decreasing.

APPLICATION FOR DYNAMIC ROAD PRICING

We applied this method to evaluate the dynamic road pricing scheme. Particularly, we think a pricing scheme which can completely eliminates the congestion: waiting time $w(t_d) = 0$ for all t_d. Then, equation (1) is changed to

$$p = \chi(t_d) + c_s(t_w - t_d) \quad . \qquad (14)$$

Where $\chi(t_d)$ is a congestion toll. Now, the problem is almost same as the former problem. We can solve $\chi(t_d)$ in the same procedure as solving $w(t_d)$ just replacing $w(t_d)$ with $\chi(t_d)$, γ with c_s, and $\gamma^+(t_w)$ with $c_s^+(t_w)$.

An example is shown in figure 5. In this figure, $c_s^+(t_w)$ is not proportional to $\gamma^+(t_w)$ because c_w is not equal for almost all travelers. For very special case where c_w is same for all travelers, no one change his/her t_d and p because $= c_s/c_w$ is proportional to c_s. This situation is occurred when each traveler's willingness to pay for reducing waiting time on the bottleneck is same. The change from $\gamma^+(t_w)$ to $c_s^+(t_w)$ means that $\chi(t_d)$ may not be proportional to $w(t_d)$ and consequently travelers' behavior may also change. If one traveler has greater c_s, he/she may change his/her t_d later and vice versa. Some traveler changes his/her schedule delay due to the change of t_d. But, we must note that we can obtain the time-dependent congestion toll $\chi(t_d)$ even when there is individual variation in time value.

These changes on $w(t_d)$, t_d, and schedule delay causes the change of their traveling cost p. From this analysis, we can hence know whether the cost p of some particular person will increase after pricing. This is important to evaluate the fairness of peak road pricing projects.

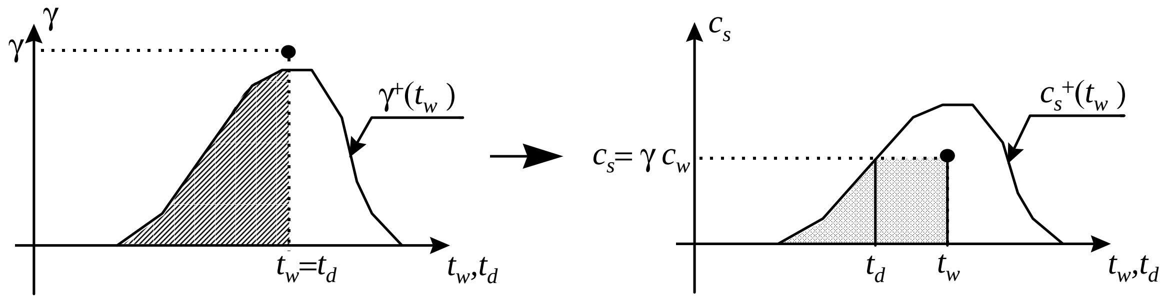

Figure 5. Travel cost is changed from the [hatched area (left)]$\times$ c_w to [dotted area (right)]. The t_d is also changed.

AN EXAMPLE

We estimated properties of travelers on the Bay-shore line of Metropolitan Expressway in Tokyo. There was a major bottleneck at Kasai JCT, where a long queue appeared every morning. We have obtained queuing delay data $w(t_d)$ (fig.6). However, we had no data for t_w, so we estimated some distribution of t_w (fig.7). We calculated γ^+ from $w(t_d)$ (fig.8) and obtained cumulative distribution of γ (fig.9). From this result, we find only the small amount of travelers with small γ. We also calculated a case of dynamic road pricing which completely eliminates congestion. We assumed distribution of c_w (fig.10), calculated distribution of c_s from this and F( ), and calculated χ (t_d) with the method mentioned the former section.

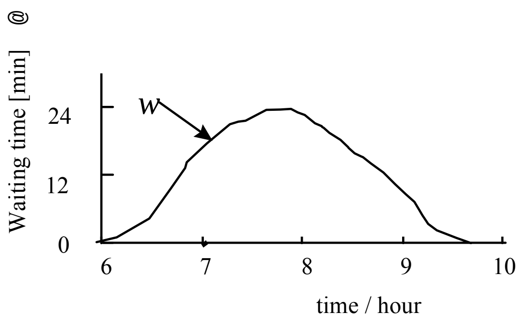

figure 6: Waiting time measured on Kasai JCT

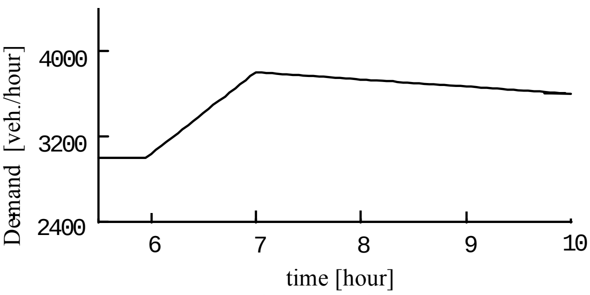

figure 7: Estimated Demand

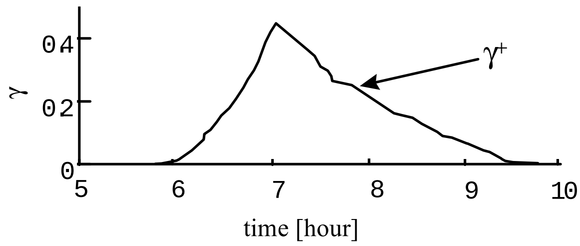

figure 8: γ^+ calculated from $w(t_d)$ on Kasai JCT

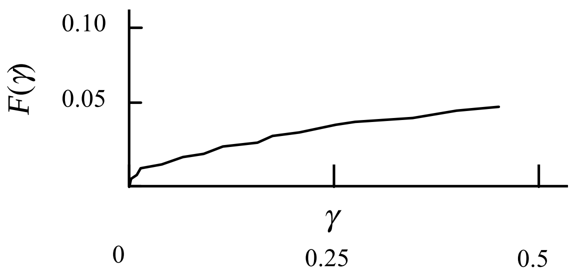

figure 9: Estimated F(γ)

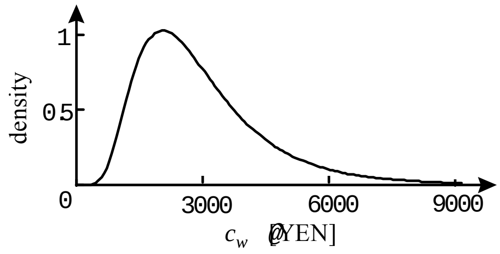

figure 10: assumed distribution of c_w

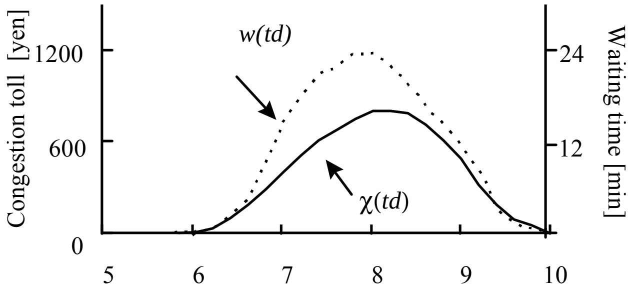

figure 11: Calculated congestion toll.

CONCLUSIONS

We have developed a new method for departure time choice problem in this research and apply this to a simple case of dynamic road pricing. This method is useful for considering dynamic road pricing policies in many situations because of the theoretical structure that is clear and visual.

We have obtained some knowledge for dynamic road pricing. The congestion toll that eliminates waiting time will not be proportional to waiting time without road pricing. After imposing the pricing, some travelers will change their choice of t_d and their travel cost will be changed. These changes are not occurred when each traveler's willingness to pay for reducing waiting time on the bottleneck is the same. Some of these results have been known[4], however, we explain these findings in visual figures and make details of changes clear.

REFERENCES

[1] V. F. Hurdle: Equilibrium Flows on Urban Freeways, *Transportation Science*, Vol.15, 1981.

[2] C. Hendrickson and E. Plank : The Flexibility of Departure Times for Work Trips, *Transportation Research*, Vol.18A, No.1, 1984.

[3] C. F. Daganzo : The Uniqueness of a Time-Dependent Equilibrium Distribution of Arrivals at a Single Bottleneck, *Transportation Science,* Vol.19, pp.29-37, 1985.

[4] G. F. Newell : The Morning Commute for Non-Identical Travelers, *Transportation Science,* Vol.21, No.2, pp.74-88, 1985.