A STUDY ON REAL-TIME SIGNAL CONTROL FOR AN OVERSATURATED NETWORK

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SUMMARY

This paper proposes a model which deals with the real-time signal control for an oversaturated network. Traffic signal, as an essential element of an urban transportation network plays a critical rule in the operation of urban street network and could not be neglected along with the movement of ITS. Over the past few decades the study on signalized intersections has been carried out by various methods, however, still it is well accepted the benefit of signal control has not been fully realized. In general, current researches on traffic signal control can be grouped into two classes from the view of flow-capacity situation, i.e. under-saturation and congestion control. The models concerning with the under-saturated flow condition serve well from isolated intersection to network control. However for a congested or over-saturated network, most of the procedures were based upon experience or simple analysis. A real-time signal control model for an oversaturated network currently does not exist.

THE LINEAR OPTIMIZATION ALGRITHM

Introduction

A signalized intersection is said to be “oversaturated” if the demand volume exceeds the capacity of the intersection. During peak demand periods at congested intersections, a traffic-actuated signal operates nearly as a pre-timed signal with a fixed value of cycle length. Under this condition, in order to maximize the total vehicular output from the oversaturated intersection or area within the congestion time period, the objective of signal timing may become one of most effectively utilizing the available green time. This paper addresses a new approach which aims at optimizing the allocation of green time under the real-time and oversaturated flow condition.

Under the over-saturated flow condition, the steady-state models break down on account of traffic flow exceeding capacity, since queues could not discharge and may persist within successive cycles, describing the dynamics of queue formation and dissipation becomes critical and a real-time formulation becomes necessary. The above part of Fig.1 illustrates a general case of the time-dependent arrival and departure process at an oversaturated approach of an signalized intersection by cumulative curves. The vertical distance between the two functions at time t_j represents the queue length at the time *jth* vehicle arrived in the system. The horizontal distance between *A(t)* and *D(t)* represents the waiting time in queue for the *jth* vehicle after it arrived in the system at time t_j. The shadow area enclosed between *A(t)* and *D(t)* during the queuing time interval (t_1, t_n) represents the total vehicle delay in queue. The objective of the linear program is to minimize the total delay within the congestion time period, subject to prevailing flow and capacity constraints.

As well known, at the signalized intersection, the departure flow follows such function as:

$$\mu(t) = \begin{cases} s(t)\, or\, v(t) & if \quad t \quad in \quad a \quad green \quad phase \\ 0 & if \quad t \quad in \quad a \quad red \quad phase \end{cases}$$

(1)

where, $\mu(t)$, *s(t)*, *v(t)* are departure flow rate, saturation flow rate, arrival flow rate respectively. Actually, the departure curve could be smoothed out by bisecting the real departure function, and therefore be presented as a smooth function as if there were no signal interruption. Accordingly, the optimization problem turns to minimize the shadow area between the arrival and the dashed curve (total delay) which is shown in the same figure.

Variables of interest

For network analysis, in general, the traffic flow is represented by directional flows with the information of destination, however, for signalized intersections the signal plan is designed based upon traffic streams. This makes it necessary to represent the traffic flow by both the directional flow and the stream. Let us denote:

$\lambda_{ijk}(t)$ = arrival rate at link (*i*,*j*) at time *t*, which travels from node *i, j* to *k;*

$\mu_{ijk}(t)$ = departure rate from link (*i*,*j*) at time *t*, which travels from node *i, j* to *k.*

respectively,

$A_{ijk}(t)$ = the cumulative arrivals at link (*i*,*j*) which travel from node *i, j* to *k* by time *t*;

$D_{ijk}(t)$ = the cumulative departures from link (*i*,*j*) which travel from node *i, j* to *k* by time *t.*

and,

$$A_{ijk}(t) = \int_t \lambda_{ijk}(t)dt \tag{2}$$

$$D_{ijk}(t) = \int_t \mu_{ijk}(t)dt \tag{3}$$

A group of streams which are accommodated in one lane group is the basic flow unit used to determine signal settings, in this paper it is defined as "stream group", let us also denote:

$\lambda_{ij}^m(t)$ = arrival rate of stream group *m* entering link (*i*,*j*) at time *t*;

$\mu_{ij}^m(t)$ = departure rate of stream group *m* departing from link (*i*,*j*) at time *t*;

$A_{ij}^m(t)$ = the cumulative arrivals of stream group *m* on link (*i*,*j*) by time *t* ($A_{ij}^m(t) = \int_t \lambda_{ij}^m(t)dt$);

$D_{ij}^m(t)$ = the cumulative departures of stream group *m* on link (*i*,*j*) by time *t* ($D_{ij}^m(t) = \int_t \mu_{ij}^m(t)dt$);

$X_{ij}^m(t)$ = the cumulative numbers left in stream group *m* by time *t*;

$$X_{ij}^m(t) = A_{ij}^m(t) - D_{ij}^m(t) \tag{4}$$

S_{ij}^m = saturation flow rate of the lane group on which stream group *m* is accommodated;

$\overline{T}_{ij}$ = free-flow travel time at link (*i*,*j*).

The linear program

Referring to the bottom of Fig.1 the objective function of the linear program could be defined as:

Minimize: $$F = \sum_{ijk}\int_t (A_{ijk}(t-\overline{T}_{ij}) - D_{ijk}(t))dt = \sum_{ijk}\int_t \{\int_u \lambda_{ijk}(u-\overline{T}_{ij})du - \int_u \mu_{ijk}(u)du\}dt \tag{5}$$

it is equivalent to:

Minimize: $$F(t) = \sum_{ijk}\left\{\lambda_{ijk}(t-\overline{T}_{ij}) - \mu_{ijk}(t)\right\} = \sum_{ijm}\left\{\lambda_{ij}^m(t-\overline{T}_{ij}) - \mu_{ij}^m(t)\right\} \tag{6}$$

Since the free-flow travel time of each link is constant and has been predetermined, and the route choice has been assumed to be given by the assignment work endogenously, each individual directional flow $\lambda_{ijk}(t-\overline{T}_{ij})$ on the link (*i*,*j*) could be obtained according to the flow conservation principle and the given route choice discipline.

Since,

$$X_{ij}^{m}(t+dt) = X_{ij}^{m}(t) + \lambda_{ij}^{m}(t)dt - \mu_{ij}^{m}(t)dt \geq 0 \qquad (7)$$

the maximum departure rate of stream group set *m* on each link (*i*,*j*) is subject to:

$$\mu_{ij}^{m}(t) \leq X_{ij}^{m}(t)/dt + \lambda_{ij}^{m}(t-\overline{T}_{ij}) \qquad (8)$$

the maximum total departure ratio at each intersection *j* is subject to:

$$\sum_{i}\sum_{p} Max_{m\in p}\left[\frac{\mu_{ij}^{m}(t)}{s_{ij}^{m}}\right] \leq \eta \qquad (9)$$

where,

p = phase number of the intersection *j*;

$\eta = 1 - \frac{L}{C}$, green time ratio of intersection *j*.

If we difine μ/s as "departure ratio", the maximum departure ratio at a signalized intersection is subjec to the green time ratio, which would be culculated by summing up the departure ratio of critical stream groups. Since, under the real-time flow condition, each stream group might be the *critical* one at present time *t*, every possible combination of stream groups at present time *t* should be considered to determine the optimal plan of allocating green time. Therefore, the left side of formula (8) is composed of every reasonable combination of stream groups. This makes it possible that this linear program could automatically change the timing plan according to present flow and saturation flow condition, to achieve the objective of maximizing the total departure within the certain period of time.

And all of the variables should be non-negative:

$$\mu_{ij}^{m}(t) \geq 0 \qquad (10)$$

The result is given in term of optimal departure flow rate which could be converted to the value of split according to given phase pattern. Then the value of $\mu_{ijk}(t)$ could be obtained based upon the First In First Out principle (The FIFO principle should be satisfied within same stream but different directional flows), it can be calculated by:

$$\frac{\lambda_{ijk}(t-T_{ijk}(t))}{\mu_{ijk}(t)}=\frac{\lambda_{ijk'}(t-T_{ijk'}(t))}{\mu_{ijk'}(t)}=\frac{\lambda_{ij}^{m}(t-T_{ij}^{m}(t))}{\mu_{ij}^{m}(t)} \tag{11}$$

where,

$T_{ijk}(t)$ = waiting time of the vehicle in the flow (i,j,k), which departs from the link (i,j) to node k at time t.

$T_{ij}^{m}(t)$ = waiting time of the vehicle in the stream group m of link (i,j).

since within the same stream group m,

$$T_{ijk}(t) = T_{ijk'}(t) = T_{ij}^{m}(t) \tag{12}$$

the $\lambda_{ijk}(t-T_{ijk}(t))$, $\lambda_{ijk}(t-T_{ijk'}(t))$, as well as $\lambda_{ij}^{m}(t-T_{ij}^{m}(t))$ could be formulated if the calculation time interval dt is less than $\overline{T}_{ij}$ for all (i,j) and m. Also since $\mu_{ij}^{m}(t)$ has been calculated by the linear program thus the $\mu_{ijk}(t)$ can be determined.

A CASE STUDY

The proposed linear program has been introduced to practice, the target area is Kichijoji-Mitaka area, which is located in the western of central Tokyo and spreads about 2 *km* from east to west and 1 *km* from north to south, it is shown on the head of Fig2. This area was first selected for the calibration of the AVENUE model (*Horiguchi, et al., 1996*). It consists of four major north-south streets and two major east-west streets. Most of the links are two-lane roads with some exceptions of four-lane roads.

Traffic congestion during morning peak period occurs constantly on Itsukaichi-kaido Street, and Inokashira-dori Street, both are caused by the traffic from west to east. Also, the north to south direction of Mitaka-dori Avenue often experiences congestion during the peak period.
The survey was carried out at 70 roadsides points within the target network, during the morning peak period from 7:00 a.m. to 10:00 a.m., the observers recorded the four large digits of the plate of all passing vehicles. The present signal parameters of each signalized intersection were recorded every fifteen minutes.

The observed data was used to extract the vehicle trajectories by matching the plate numbers and vehicle types between two neighboring roadside point. Travel time are calculated by subtracting the passing time. The extracted vehicle trajectories were then used to derive OD demands and turning ratio on each link. Eliminating the centroids and non-signalized intersections, the road network for formulation is illustrated on the bottom of Fig.2. The values of link length are shown directly in the middle of the link, with the unit of *meter*. All links are two-way links except for link *(21,22)* and link *(22,23)*.

The total target network is divided into three parts, i.e. sub-networks, according to present signal design, each shares one fixed cycle length during the study period, they could be formulated separately for practicing the model developed in this paper. The purpose for this practice is to propose new split design to optimize the capacity of the sub-network during the morning peak period. The value of signal parameters of offset and cycle length were taken from the present design. In this paper, we choose the sub-network one to present the calculation result, the physical configuration and phase pattern of each intersection is shown in Fig.3. As an example, table 1 shows the optimized split values during 8:00~8:15 a.m. time period.

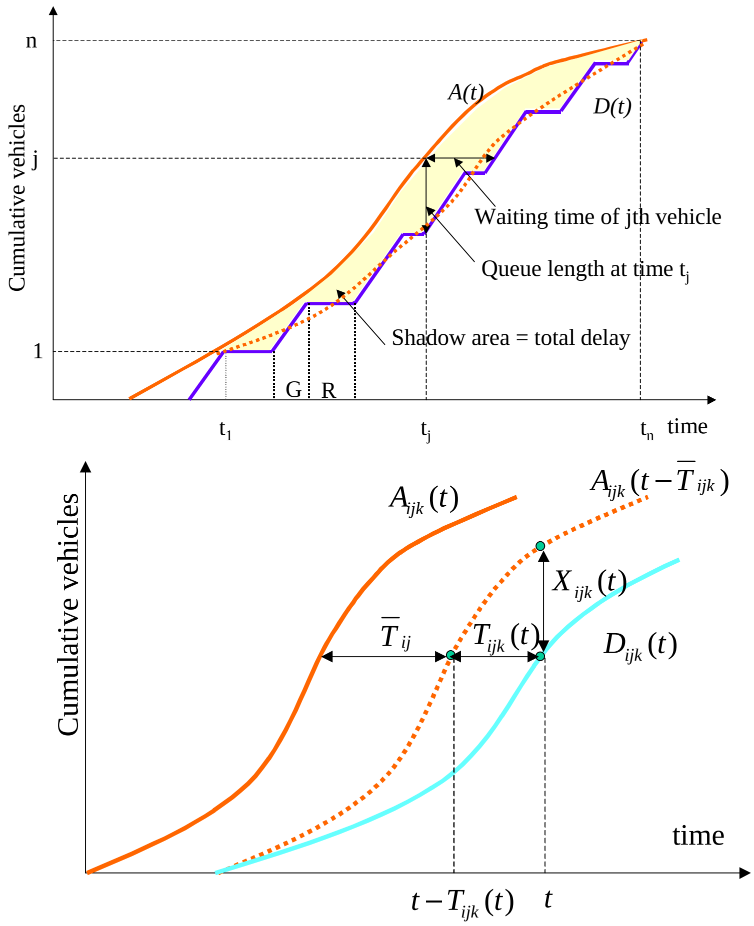

Figure 1: graphical illustration

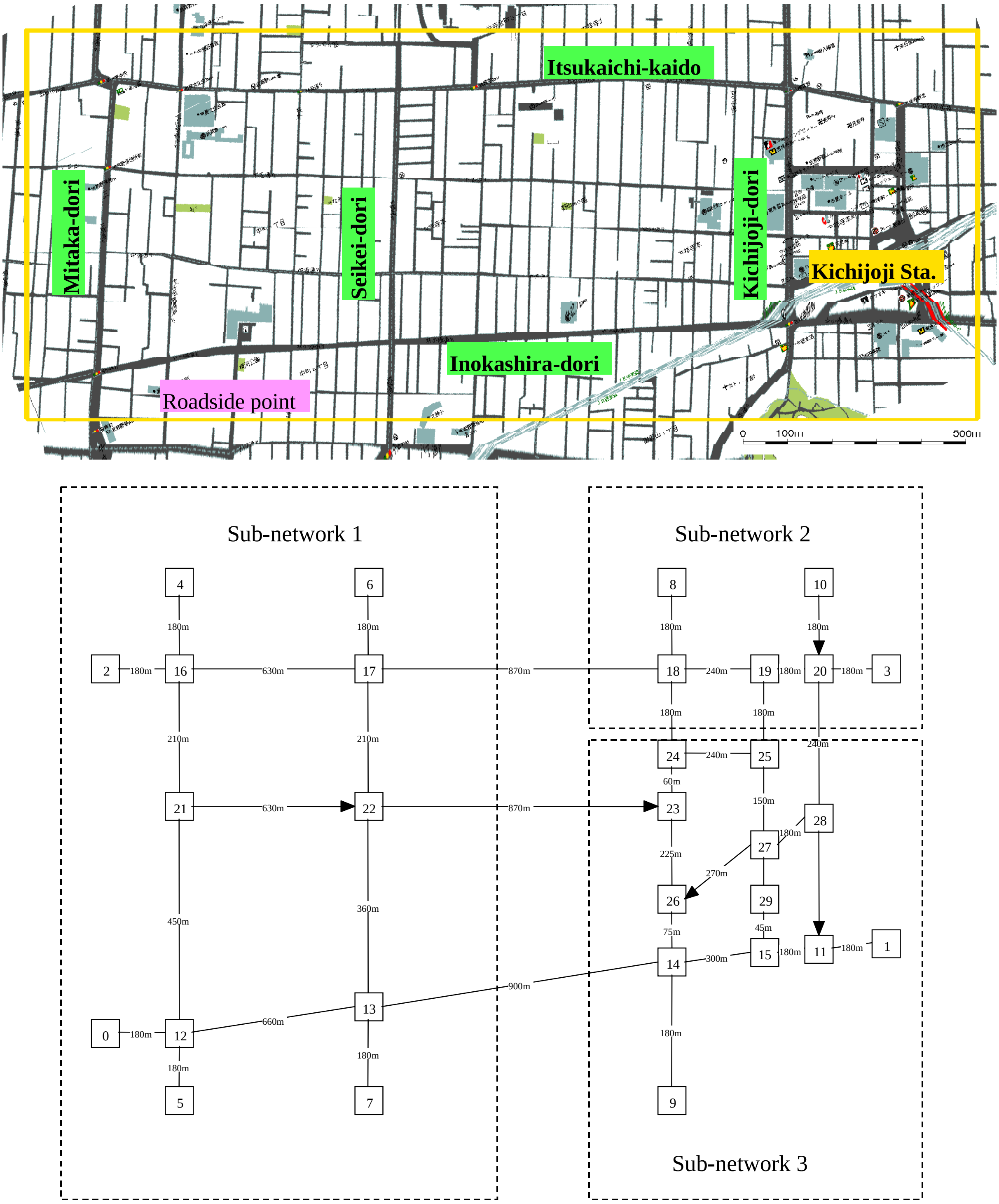

Figure 2: the network

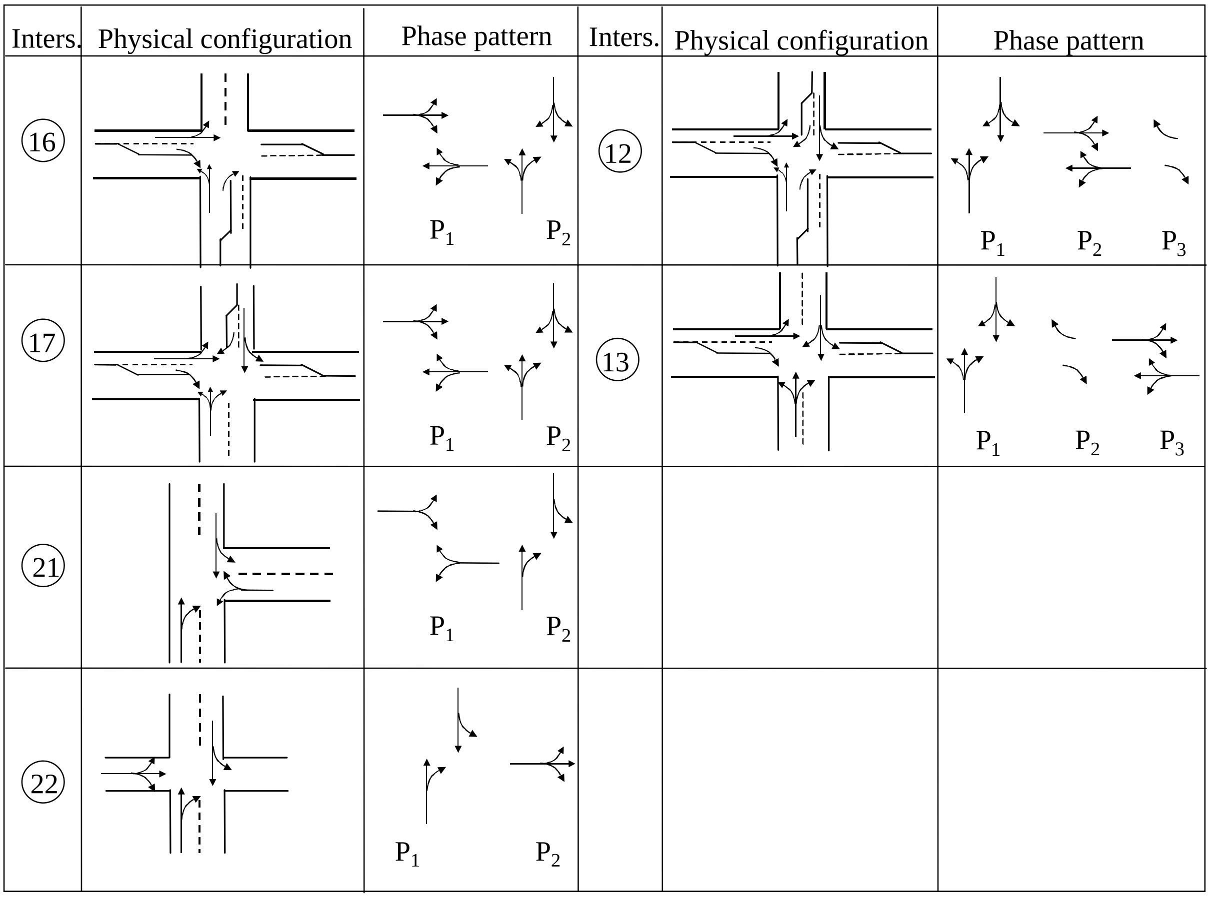

Figure 3: intersection configuration and phase pattern

Table1 : formulation result

intersection	cycle length (sec)	lost time (sec)	relative offset (sec)	base intersection	split (%)	green time (sec)
16	120	12	64	17	P1=0.44	G1=44
					P2=0.56	G2=64
17	120	12	36	18	P1=0.66	G1=71
					P2=0.34	G2=37
21	120	12	0	12	P1=0.78	G1=84
					P2=0.22	G2=24
22	120	12	0	12	P1=0.59	G1=64
					P2=0.41	G2=44
12	120	12	-42	13	P1=0.67	G1=72
					P2=0.27	G2=29
					P3=0.06	G3=7
13	120	12	66	14	P1=0.40	G1=43
					P2=0.07	G2=8
					P3=0.53	G3=57

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