A Theoretical Analysis on Dynamic Marginal Cost Pricing

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Abstract

This study extends the static marginal cost analysis to the dynamic one so that the dynamic bottleneck phenomena are properly included in its supply curve. Conventionally in the economic field, the equilibrium analysis of demand-supply has been studied and the marginal cost pricing strategy was proposed so as to maximize consumer's surplus. However, the static analysis does not well consider the time-dependent queue evolution in the supply function. On the other hand, in traffic engineering field, although the time-dependent queueing analysis has been extensively studied, the demand function has not been dealt with in most analysis. This study attempts to extend the demand-supply analysis to incorporate dynamic queueing phenomena. For the problem without departure time choice, it is shown that dynamic marginal cost is equivalent to the duration of congestion. For the problem with departure time choice, the demand-supply analysis is combined with the existing theory of departure time choice with time constraints at destinations.

1. INTRODUCTION

This study extends the current static marginal cost analysis to the dynamic one so that the dynamic bottleneck phenomena are properly included in its supply curve. Conventionally in the economic field, the equilibrium analysis of demand-supply has been studied using demand as well as supply functions and the marginal cost pricing strategy was proposed so as to maximize consumer's surplus. However, the static analysis does not well consider the time-dependent queue evolution in the supply function. On the other hand, in traffic engineering field, although the time-dependent queueing analysis has been extensively studied given certain amount of demand, the demand function has not been explicitly dealt with in most analysis. Thus, this study attempts to extend the equilibrium demand-supply analysis so as to incorporate dynamic queueing phenomena.

2. REVIEW OF STATIC MARGINAL COST ANALYSIS

Let us quickly review the derivation of the marginal cost pricing in the static framework. The demand and supply functions are first defined as follows:

$\rho(p)$ = demand generated at trip cost p (1)
$S(x)$ = trip cost when demand is x (2)

These functions are described in Fig.1 and the intersection of two curves is the equilibrium point at demand x^*. The consumer's surplus, the shaded are in Fig.1, is then written as

$$F = \int_0^x \rho^{-1}(y)dy - x \cdot S(x)\ . \qquad (3)$$

Taking derivative with respect to demand x yields

$$\rho^{-1}(x) = \frac{d}{dx}\{x \cdot S(x)\} = MC(x)\ . \qquad (4)$$

This implies that the right hand side of the marginal cost $MC(x)$, the derivative of total cost regarding x, must be equal to the inverse of demand function $\rho^{-1}(x)$.

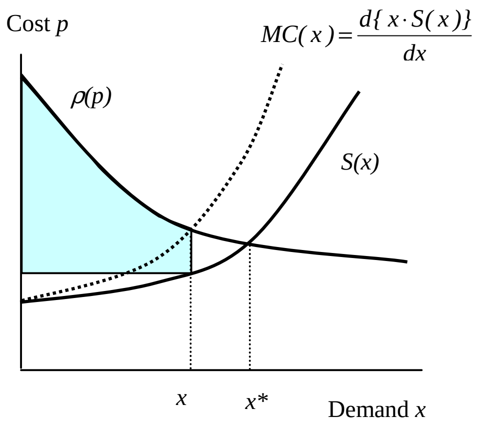

Fig.1 Static Demand-Supply Equilibrium

3. DYNAMIC BOTTLENECK PHENOMENA

Queue evolution at a bottleneck is basically time-dependent phenomena, which cannot be described by the above static analysis. Let us consider a single bottleneck. If traffic demand rate exceeding the bottleneck capacity μ enters, a queue would grow as shown in Fig.2. The arrival and departure rates $\lambda(t)$ and $\mu(t)$, and waiting time at the bottleneck $w(t)$ are defined:

$\lambda(t)$ = arrival rate at the bottleneck at time t,
$\mu(t)$ = departure rate from the bottleneck at time t,
$w(t)$ = waiting time of a user arriving at the bottleneck at time t.

And the cumulative arrival and departures are also defined as follows:

$$A(t) = \int_0^t \lambda(t)dt \quad (5)$$

$$D(t) = \int_0^t \mu(t)dt \quad (6)$$

Note that in Fig.2, the cumulative curves are drawn based on the point queue concept. Under the FIFO queue discipline, the horizontal distance between $A(t)$ and $D(t)$ shows the waiting time $w(t) = D^{-1}(A(t)) - t$ as shown in the figure. From this figure, the waiting time at time t, $w(t)$, is clearly depends upon the history of $\lambda(t)$ until time t. Waiting time $w(t)$, which corresponds to trip cost, therefore cannot be simply a function of the demand rate at that time t, but $w(t)$ may change if $\lambda(t')$, $t' < t$ varies.

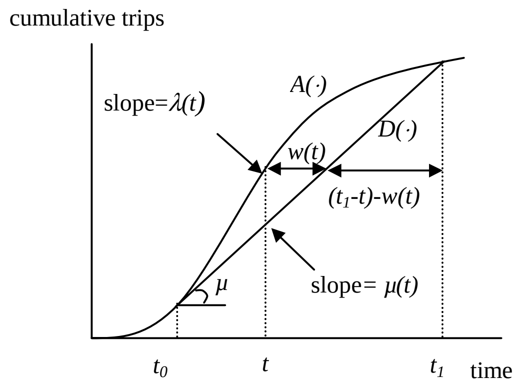

Fig.2 Cumulative Arrival and Departure at a Bottleneck

4. EXTENSION TO DYNAMIC MARGINAL COST ANALYSIS

4.1. Dynamic Marginal Cost

Let's consider the dynamic marginal cost by adding the time axis. As shown in Fig.3, cost $p(t)$ is defined as the cost of a user arriving at the bottleneck at time t and written as a function of $w(t)$: $p(t)=f_w\{w(t)\}$. On the other hand, the demand during t~$t+dt$ is written as $x(t)=\lambda(t)dt$. Similar to the static analysis, the demand function is defined:

$$\rho(p(t),t) = x(t), \quad (7)$$

$$\rho^{-1}(x(t),t) = p(t). \quad (8)$$

The demand function $\rho(p(t),t)$ is assumed given for all t, and the upper figure of Fig.3 shows that $\rho(p(t),t)$ are changing over time. This time-dependent demand function is introduced to reflect the fact that the generated demand rate may change over time, for instance peak and off-peak, even if the same amount of cost is imposed to users. At time t, waiting time $w(t)$ is evaluated from the cumulative curves as in the lower figure. Then, the cost at time t, $p(t)=f_w\{w(t)\}$, determines the generated demand of $\rho(f_w\{w(t)\},t)=\lambda(t)dt$ during t~$t+dt$. The demand rate $\lambda(t)$ must be equal to the slope of $A(t)$ at time t. This equilibrium is here called as "*demand-supply equilibrium*".

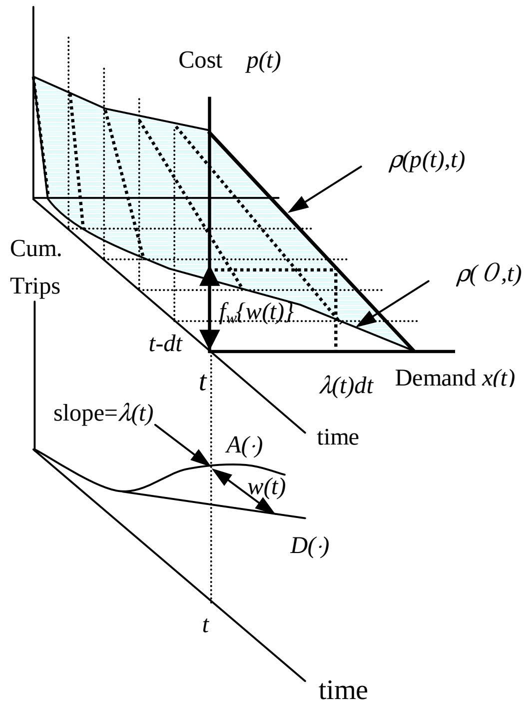

Fig.3 Time-Dependent Demand Plane and Queue Evolution

Employing the discrete time increment of dt, the consumer's surplus F is written as

$$\begin{aligned} F &= F_1 - F_2 \\ &= \sum_t \int_0^{\lambda(t)dt} \rho^{-1}(x,t)dx - \sum_t f_w\{w(t)\}\lambda(t)dt \,. \end{aligned} \tag{9}$$

The optimality condition to maximize F is obtained by taking derivative with respect to $x(t)=\lambda(t)dt$. The derivative of the first term F_1 is

$$\begin{aligned} \frac{\partial F_1}{\partial \lambda(t)dt} &= \rho^{-1}(\lambda(t)dt,t) + \sum_{u=t+1}^{t_1} \frac{\partial \lambda(u)}{\partial \lambda(t)} \rho^{-1}(\lambda(u)dt,u) \\ &= \rho^{-1}(\lambda(t)dt,t) \end{aligned} \tag{10}$$

Since $\lambda(u)$, $u>t$ can be controlled independent of $\lambda(t)$ at time t, the second term on the right hand side, $\partial\lambda(u)/\partial\lambda(t)$, becomes zero. On the other hand, the derivative of F_2 means $MC(t)$ because it is the derivative of the total cost:

$$\begin{aligned} MC(t) &= \frac{\partial F_2}{\partial \lambda(t)dt} = \frac{\partial \sum_t f_w\{w(u)\} \cdot \lambda(u)dt}{\partial \lambda(t)dt} \\ &= \sum_{u=t}^{t_1} \frac{\partial \lambda(u)dt}{\partial \lambda(t)dt} f_w\{w(u)\} + \sum_{u=t}^{t_1} \frac{df_w\{w(u)\}}{dw(u)} \frac{\partial w(u)}{\partial \lambda(t)} \lambda(u) \\ &= f_w\{w(u)\} + \sum_{u=t}^{t_1} \frac{df_w\{w(u)\}}{dw(u)} \frac{dt}{\mu} \lambda(u) \end{aligned} \tag{11}$$

However, the above result is applicable only when the bottleneck is busy, $w(t)>0$ or $\lambda(t) \geq \mu$. When the bottleneck is idle just like before t_0, marginal cost $MC(t)$ is obviously equal to $f_w\{0\}$, since $w(t)$ cannot be affected by the demand change and consequently the second term of the right hand side becomes zero. As a whole, the optimality condition is summarized as follows:

$$\rho^{-1}(\lambda(t)dt,t) = MC(t) = \begin{cases} f_w\{w(u)\} + \sum_{u=t}^{t_1} \frac{df_w\{w(u)\}}{dw(u)} \frac{dt}{\mu} \lambda(u), & w(t)>0 \text{ or } \lambda(t) \geq \mu \\ f_w\{0\}, & \text{otherwise} \end{cases} \tag{12}$$

For a special case, if the waiting cost function is linear:

$$f_w\{w\}=bw, \qquad b>0, \tag{13}$$

the marginal cost is simply written as

$$MC(t) = \begin{cases} b(t_1 - t), & \text{if } w(t)>0 \text{ or } \lambda(t) \geq \mu \\ 0 & \text{, otherwise,} \end{cases} , \tag{14}$$

where t_1-t means the duration of the congestion after time t because time t_1 is the queue vanishing time.

Since user cost is $f_w\{w(t)\}$, the dynamic toll while the bottleneck is busy is equal to the first term of Eq.(11):

$$\sum_t^{t_1} \frac{df_w\{w(u)\}}{dw(u)} \frac{dt}{\mu} \lambda(u) = b\{t \;\; - t - w(t)\}, \quad \text{for linear waiting cost function.} \tag{15}$$

If you apply the marginal cost pricing to users in Fig.2, no toll should be imposed before time t_0, because the bottleneck is idle and $MC(t)=f_w(0)$. At time t_0, arrival rate $\lambda(t)$ becomes equal to capacity μ. If you impose the toll determined from Eq.(15), clearly the arrival rate would be below the capacity; that is, the dynamic toll is too much. The best pricing is to impose toll which controls the arrival rate just equal to its capacity. That is, from Eq.(12), $MC(t)$ is discontinuous when $w(t)=0$ as well as $\lambda(t)=\mu$ such as time t_0. Namely, when the demand increases by one unit, $MC(t)$ is equal to the upper value of Eq.(12); on the other hand, when it decreases, $MC(t)$ jumps to the lower value. Apparently, as the idle bottleneck due to the pricing does not improve the consumer's surplus, the best strategy for this situation is to impose the toll so as to control the arrival rate just equal to its capacity.

Therefore, during some time after time t_0, the arrival rate must be controlled at capacity μ. Meanwhile, the potential demand would become sufficient so that demand larger than μ could be generated even if toll of Eq.(15) is fully imposed.

4.2. An Example

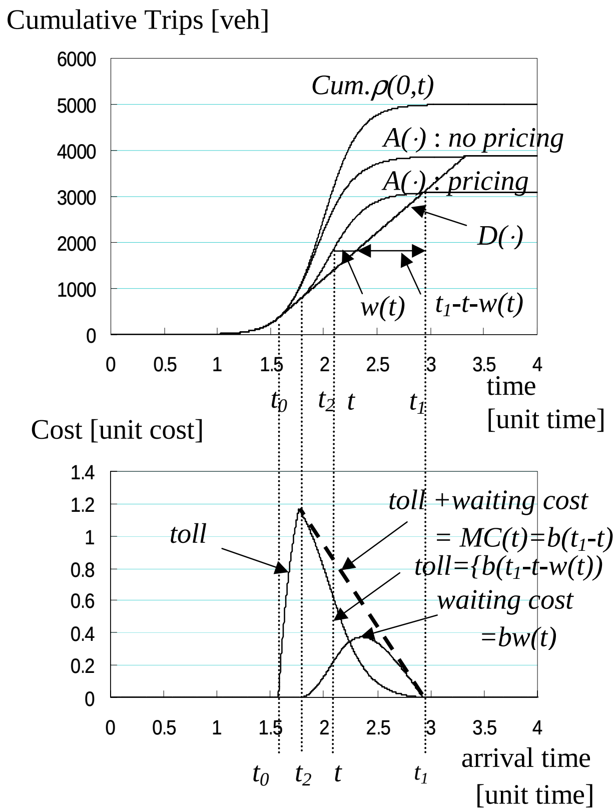

Fig.4 Marginal Cost Pricing at a Single Bottleneck

Let us consider a bottleneck with its capacity of 2000[veh/unit time]. The demand function $\rho^{-1}(x,t)$, which gives demand x[veh] during a short interval of dt, is assumed in the following linear form:

$$\rho^{-1}(x,t) = -\frac{a_0}{\rho(0,t)}\{x-\rho(0,t)\} \text{ [unit cost],}$$

where $\rho(0,t)$ as in Fig.3 means the maximum demand when the user cost is zero and its cumulative value is shown in the upper figure of Fig.4. Also, the waiting cost function is assumed linear:

$f_w\{w\}=bw$, $\quad b=1.0$ [unit cost/unit time].

Using these linear functions, the cumulative arrival curve with the marginal cost pricing can be easily drawn in the following manner. Suppose that we have evaluated $A(t)$ until time t and try to extend $A(t)$ thereafter.

Step 1 : Assume queue vanishing time t_1.

Step 2 : From $A(t)$ until time t, $D(t)$ can be determined until time $t+w(t)$. From $A(t)$ and $D(t)$, waiting time of a user arriving at time t, $w(t)$, is evaluated and hence the dynamic toll is also determined as $b\{(t_1-t)-w(t)\}$.

Step 3 : From the dynamic toll and $w(t)$, demand generated during t～$t+dt$ is determined from the given demand function: $\lambda(t)dt=\rho(b(t_1-t),t)$. Then, the arrival curve is extended up to time $t+dt$: $A(t+dt) = A(t) + \lambda(t)dt$.

Step 4 : Update time t as $t+dt$, and return to Step 2 if time $t+dt$ is still within the study period.

Step 5 : From $A(t)$ drawn, find queue vanishing time t_1'. If t_1' is equal to the assumed time

t_1, end; otherwise return to Step 1. In Step 1, t_1 must be modified using its characteristics that t_1' is monotonically decreasing with respect to t_1 assumed in Step 1.

Fig.4 shows the cumulative curves and the time-dependent toll. We notice that queue starting time t_0 does not change before and after pricing. This is due to no pricing before time t_0 because of no sufficient demand. As explained above, from time t_0 to t_2, the dynamic toll is adjusted so that the generated demand rate is just equal to the capacity. After time t_2, since the potential demand becomes larger and the amount of $b\{(t_1-t)-w(t)\}$ obtained from Eq.(15) is fully imposed until the queue vanishes at time t_1. As shown in the dashed line, the marginal cost from Eq.(14) is linearly decreasing from t_2 to t_1: $MC(t) = b(t_1-t)$. In this example, the consumer's surplus increases from 3095 [veh*unit cost] to 3663 [veh*unit cost] by the pricing.

5. ANALYSIS WITH DEPARTURE TIME CHOICE

Discussion so far has been limited within cases where a user has only one choice of whether he/she makes a trip. However, normally users would have other choices as well such as modes, departure times, etc. In this section, among these choices, we would like to add departure time choice into the dynamic marginal cost analysis.

5.1. Departure Time Choice Problem with Time Constraint

Mainly from 1980s, several studies on departure time choice have been reported mostly for trips with desired arrival times at destinations (Vickrey(1969), Hendrickson (1981), and Kuwahara (1987)). The typical application was morning commute trips, in which commuters have time constraints of their work schedules. Trips of other trip purposes may more or less have some time constraints at destinations; for instance in recreational trips, travelers might have approximate desired arrival times at their home. Thus, we would like to consider the previous research to include departure time choice to the marginal cost analysis.

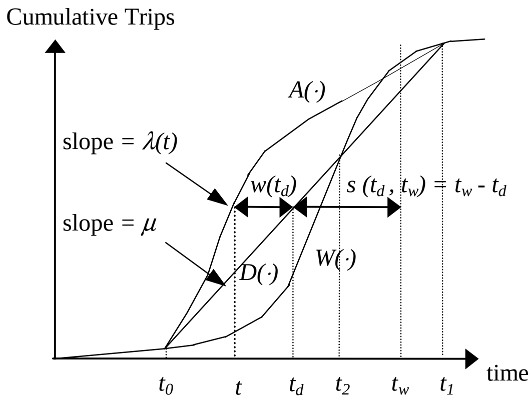

Fig.5 Queue Evolution at a Single Bottleneck, and Waiting Time and Schedule Delay

Let's review the departure time choice problem at a single bottleneck with its capacity μ. Everyone is assumed to pass through the bottleneck to reach the destination. Since each user has the desired arrival time at the destination, the trip cost of a user consists of waiting cost at the bottleneck and schedule cost associated with time difference between the actual and desired arrival times. The trip cost of a user with desired arrival time t_w is therefore written as

$$p(t_d , t_w) \quad = f_w\{w(t_d)\} + f_s\{s(t_d, t_w)\} \qquad (16)$$

t_d = departure time from the bottleneck
t_w = desired arrival time at the destination (given)
$p(t_d , t_w)$ = trip cost of a user with desire arrival time t_w departing from the bottleneck at

time t_d.

$w(t_d)$ = waiting time at the bottleneck for a user departing at time t_d. (In the previous sections, waiting time is defined by referring arrival time at the bottleneck. However, under FIFO, waiting time can be defined also with respect to departure time from the bottleneck.)

$s(t_d , t_w)$ = schedule delay for a user with desired arrival time t_w departing from the bottleneck at time $t_d = t_w - t_d$

$f_w\{w\}$ = waiting cost function which converts waiting time to the cost

$f_s\{s\}$ = schedule cost function which converts schedule delay to the cost

A user is assumed to travel from the origin to the bottleneck at a static travel speed, wait at the bottleneck, and again travel to the destination at a static travel speed. Therefore, only waiting time and schedule delay are time-dependent. Thus, for our convenience, the desired arrival time at the destination can be switched to the desired departure time from the bottleneck, since we can easily obtain the desired departure time by subtracting static travel time from the bottleneck to the destination from the desired arrival time at the destination. The schedule delay can be therefore written as $s(t_d , t_w) = t_w - t_d$. Each user is assumed to choose the best departure time from the bottleneck so as to minimize his/her own trip cost.

An important property of this problem is the First In First Work (FIFW) discipline in which the order of arrival times at the bottleneck is the same as the order of desired arrival times t_w's, provided that schedule cost function $f_s\{s\}$ is convex in s (Smith (1981) and Daganzo (1985)). Since, under FIFW together with FIFO, $A(t)=D(t_d)=W(t_w)$ is valid, where $W(t_w)$ is the cumulative users with desired arrival times before t_w. Arrival time at the bottleneck t and departure time from the bottleneck t_d can be written as a function of desired arrival time t_w:

$$t_d(t_w) = D^{-1}(W(t_w)), \qquad t(t_w)=A^{-1}(W(t_w)). \tag{17}$$

Also, $w(t_d)$, $\lambda(t)$, and $p(t_d, t_w)$ can be described with respect to t_w:

$$w(t_w) = w(t_d(t_w)),\ \lambda(t_w) = \lambda(t(t_w)),\ \text{and}\ p(t_w) =p(t_d(t_w),t_w)= f_w\{w(t_w)\} + f_s\{t_w-t_d(t_w)\}. \tag{18}$$

It has been also known that the following "*temporal equilibrium*" condition must be satisfied at the equilibrium situation (Kuwahara et.al.(1987)):

$$\begin{aligned}\frac{dp(t_d(t_w),t_w)}{dt_w} &= \frac{\partial p(t_d(t_w),t_w)}{\partial t_w} + \frac{\partial p(t_d(t_w),t_w)}{\partial t_d}\cdot\frac{dt_d(t_w)}{dt_w} \\ &= f_s'\{s(t_d(t_w),t_w)\} = f_s'\{s(t_w)\}\ , \quad (\because \frac{\partial p(t_d(t_w),t_w)}{\partial t_d} = 0),\end{aligned} \tag{19}$$

where $f_s'\{s\} = \dfrac{d f_s\{s\}}{ds}$.

By solving the above differential equation, the user cost at desired arrival time t_w, $p(t_w)$, can be written simply as a function of schedule delay:

$$p(t_w) = \int_{t_0}^{t_w} f_s'\{s(t)\}dt\ . \tag{20}$$

5.2. Demand Function and Dynamic Marginal Cost

With departure time choice, we have to simultaneously establish "*temporal equilibrium*" in addition to "*demand-supply equilibrium*" discussed in section 4.

5.2.1 Dynamic Marginal Cost

For the problem with departure time choice, the distribution of desired arrival times, $W(t_w)$, can be considered as the generated demand, since the cumulative arrival curve A(t) is obtained as the result of departure time choices given $W(t_w)$. Therefore, defining $\eta(t_w) = dW(t_w)/dt_w$, the demand function is written as below with respect to t_w:

$$\rho(p(t_w), t_w) = \eta(t_w)dt_w \tag{21}$$

$$\rho^{-1}(\eta(t_w)dt_w, t_w) = p(t_w) \tag{22}$$

The consumer's surplus F is then written

$$\begin{aligned} F &= F_1 - F_2 \\ &= \sum_{t_w}\int_0^{\eta(t_w)dt_w} \rho^{-1}(x,t_w)dx - \sum_{t_w} p(t_w)\eta(t_w)dt_w \end{aligned} \tag{23}$$

The optimality condition is obtained by differentiating with respect to demand $\eta(t_w)dt_w$:

$$\rho^{-1}(\lambda(t_w)dt_w, t_w) = MC(t_w) = \begin{cases} p(t_w) + \sum_u \dfrac{\partial p(u)}{\partial \eta(t_w)}\eta(u), \; w(t_w) > 0 \text{ or } \lambda(t_w) \geq \mu \\ p(t_w), \qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{24}$$

The second term on the right hand side cannot be generally written in the explicit form, since if demand $\eta(t_w)dt_w$ increases by one unit at time t_w, $W(t_w)$ as well as $A(t)$ and $D(t)$ would shift for an entire time period as shown in dashed lines in Fig.6. Consequently user cost $p(t_w)$ would also change for all time t_w.

5.2.2. Constant Desired Arrival Time for Everyone

Although the evaluation of the right hand side of Eq.(24) is difficult in general, we could relatively easily analyze a special case where everyone has the same desired arrival time. Since everyone has the same desired time, we cannot distinguish users and therefore everyone has the same cost under the temporal equilibrium condition. From Eq.(20), we see that the cost for everyone is given by $f_s\{t_w\text{-}t_0\}$. As shown in Appendix, with this assumption, marginal cost $MC(t_w)$ is explicitly written as

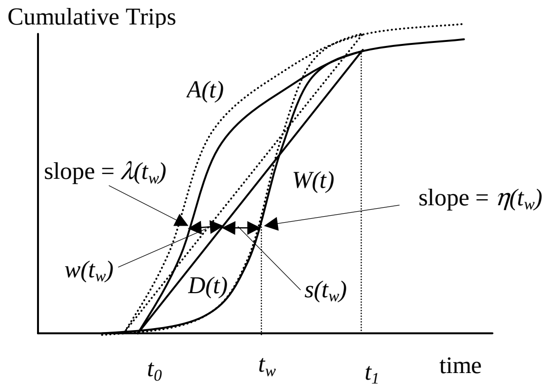

Fig.6 Impact on $A(t)$ and $D(t)$ by Demand shift

$$MC(t_w) = f_s\{t_w - t_0\} + \frac{N}{\mu}\left[\frac{1}{f_s'\{t_w - t_0\}} - \frac{1}{f_s'\{t_w - t_1\}}\right]^{-1}, \text{ if } w(t_w) > 0 \text{ or } \lambda(t_w) \geq \mu \tag{25}$$

where $N = W(t_1) - W(t_0)$.

The second term corresponds to the dynamic toll, which is the same for everyone.

For the more simplification, let us consider the following linear schedule cost function:

$$f_s\{ s \} = \begin{cases} c_1 \cdot s, & s \ge 0, \\ -c_2 \cdot s, & s < 0, \end{cases} \qquad c_1 > 0 \text{ and } c_2 > 0. \tag{26}$$

Then, $MC(t_w)$ becomes,

$$MC(t_w) = p(t_w) + \frac{N}{\mu}(\frac{1}{c_1} + \frac{1}{c_2})^{-1}, \qquad \text{if } \ w(t_w) > 0 \ \text{ or } \ \lambda(t_w) \ge \mu \cdot \tag{27}$$

On the other hand, from Eq.(20),

$$p(t_w) = (\frac{1}{c_1} + \frac{1}{c_2})^{-1} \frac{N}{\mu} \quad . \tag{28}$$

Thus, $MC(t_w)=2p(t_w)$ with the linear function, which means the toll is $p(t_w)$ same for everyone.

Under this simplification, since everyone has the same trip cost as well as the same amount of toll, the demand-supply balance can be summarized in one graph as shown in Fig.7. From Eq.(27) and (28), the user cost and the marginal cost can be written as functions of the demand *N*: *p(N)* and *MC(N)*. The *MC(N)* has a slope twice as much as one for *p(N)*. The optimal demand-supply equilibrium is the intersection of demand curve $\rho(p(N))$ with *MC(N)* at the marginal cost pricing of *p(N*)*.

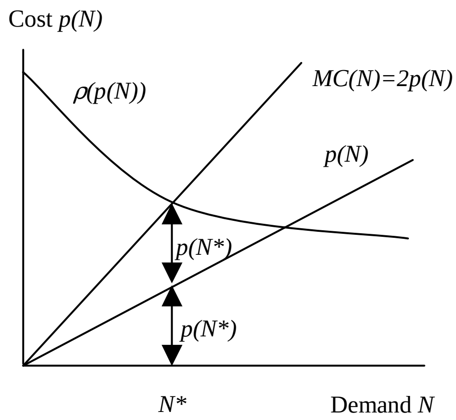

Fig.7 Demand-Supply Equilibrium when Everyone has the Same Desired Arrival Times (Linear Schedule Cost Function)

5.3. Discussion

Based on the above analysis under several assumptions for simplification, let us discuss the implications.

(1) Addition of the static cost

We have eliminated the static travel cost such as static portion of travel time and static tolls from our analysis. If you would like to include them, you simply add the static cost to the dynamic cost we analyzed. For instance in Fig.7, we may add some constant cost to user cost *p(N)* as well as *MC(N)*.

(2) Different desired arrival time for each user

In reality, $W(t_w)$ would be distributed over some time interval as in Figures 5 and 6. For the distributed $W(t_w)$, user cost depends upon

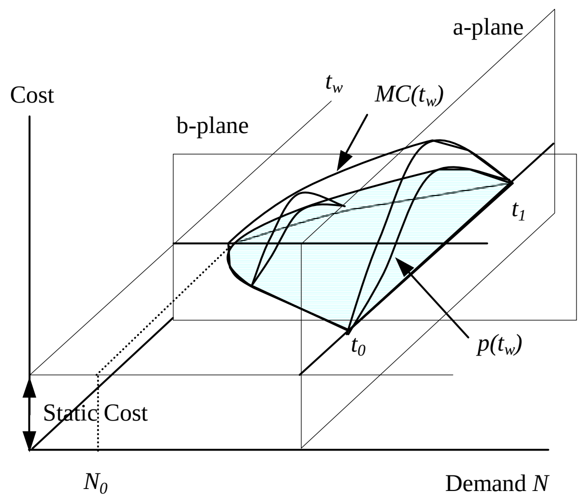

Fig.8 Three-Dimensional Illustration of User and Marginal Costs

his/her desired arrival time t_w, and we cannot summarize the demand-supply equilibrium within one graph as Fig.7. Instead, $p(t_w)$ and $MC(t_w)$ would form planes as shown in Fig.8, which includes the static cost mentioned above. Fig.9 illustrates how user cost changes in relation to the desired arrival time t_w with fixed demand N (a-plane in Fig.8). From Eq.(20), queueing delay starts from time t_0, reaches to its maximum value at t_w for zero schedule delay, and finally vanishes at time t_1. If the demand N gets smaller, times t_0 and t_1 would approach each other and eventually coincide. Namely, in Fig. 8, no queue forms when the demand is less than N_0.

On the other hand, Fig.10 illustrates how $p(t_w)$ and $MC(t_w)$ vary depending on demand N by with fixed desired arrival time t_w (b-plane in Fig.8). Note that Fig.7 is a special case of Fig.10 when t_w is the same for everyone. Although Fig.10 looks similar to the demand-supply curves under the static framework as in Fig.1, the implication is quite different. First, the shapes of $p(t_w)$ and $MC(t_w)$ depend upon t_w. Second, the location N_0 where $p(t_w)$ starts increasing depends on the distribution of t_w, $W(t_w)$. For the same desired arrival time, $W(t_w)$ has the infinite slope at time t_w. Hence, a queue must form even for a very small demand N, which is the reason why $p(N)$ start increasing for $N>0$. However, when t_w's are distributed over some time period, it would be quite possible that no queue forms for small $N < N_0$. Third, the tendency of increasing $MC(t_w)$ for $N > N_0$ is related to the schedule cost function as well as times t_0 and t_1 as explained in Eq.(25) and (27). For a linear schedule cost function with the constant desired arrival time, we have seen that $p(t_w)$ and $MC(t_w)$ become linear as in Eq.(27) and (28). But, for a general case, explicit forms of them cannot be described.

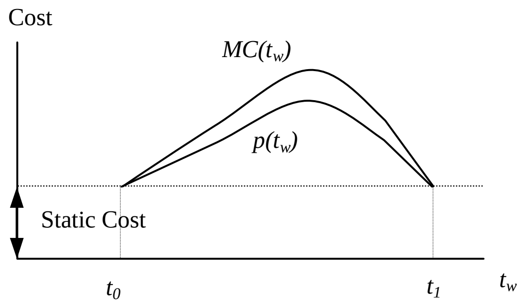

Fig.9 User and Marginal Cost in Profile (Fixed total demand N)

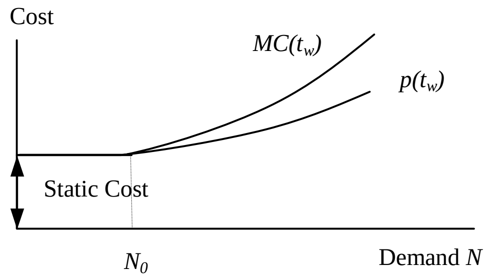

Fig.10 User and Marginal Costs in Relation to Total Demand N (Fixed t_w)

(3) Converting waiting time to toll

Even with the dynamic marginal cost pricing, a queue must form to establish the temporal equilibrium for sufficiently large demand. The waiting time is required to compensate the schedule delay changing over time for the temporal equilibrium. However, from theoretical point of view, we could impose additional dynamic toll equivalent to the waiting time value to eliminate the queueing delay. The temporal equilibrium would be maintained after the additional toll, since users pay the same amount of cost as the waiting cost. (Note that for the problem without departure time choice discussed in section 4, the conversion of waiting cost to toll is impossible. Since the demand rate is the slope of arrival curve $A(t)$, the $A(t)$ and $D(t)$ do not necessarily coincide each other after the conversion under the demand-supply equilibrium.)

(4) Demand-Supply equilibrium

Figures 8, 9, and 10 are drawn if schedule cost function $f_s\{s\}$ as well as $W(t_w)$ are assumed, since user cost $p(t_w)$ under the temporal equilibrium can be determined. However, for demand-supply equilibrium, generated demand from demand function $\rho(p(t_w),t_w)$ must be equal to $\eta(t_w)dt_w$ which is evaluated from the assumed $W(t_w)$. For the problem without departure time choice, the user cost at time t depends only on arrival rate $\lambda(t)$ before t under FIFO. As explained in section 4.2, we can determine generated demand rate $\lambda(t)$, which satisfies the demand-supply equilibrium, sequentially from the earlier time. However, cost $p(t_w)$ here depends upon the demand rate of the entire time period. An efficient algorithm to find the demand-supply equilibrium is not proposed yet.

6. SUMMARY AND FUTURE RESEARCH NEEDS

This research analyzes the dynamic marginal cost by combining the demand-supply analysis in economics with the time-dependent queueing analysis in traffic engineering. Main conclusions are listed below:
(1) For the problem without departure time choice, the dynamic marginal cost at time t, $MC(t)$, is equal to the duration of congestion after time t.
(2) Through a simple example, we show that the consumer's surplus can be improved by introducing the dynamic marginal cost pricing.
(3) For the problem with departure time choice, the demand-supply analysis is combined with the existing theory of departure time choice. When desired arrival time t_w is distributed over time, $MC(t_w)$ cannot be explicitly written. However, under the simplification that everyone has the same desired arrival time, user cost $p(t_w)$ and marginal cost $MC(t_w)$ are explicitly described as a function of the total generated demand during the congested time period.
(4) Waiting time at a bottleneck can be converted to the dynamic toll for the problem with departure time choice.

Several future research needs are summarized:
(1) This research mainly focuses on the supply function so that it reflects dynamic bottleneck phenomena. However, for the demand-supply analysis, the demand function must be evaluated. The demand function we have introduced here forms a three-dimensional plane as in Fig.3, and therefore it would be quite difficult to evaluate the shape.
(2) In this analysis, we assume that the generated demand at a certain time depends only on user cost at that time. Several different definitions on the demand function, however, could be possible. For instance, one can propose that the demand may depend on the average user cost for some time period or for the entire study period. As mentioned in (1) above, the definition and evaluation of the demand function are the major research needs.
(3) For the problem with departure time choice, we should propose an algorithm to find the solution which satisfies both the temporal as well as demand-supply equilibrium.

APPENDIX

The $\dfrac{\partial p(u)}{\partial \eta(t_w)}$ in Eq.(24) is written as

$$\frac{\partial p(u)}{\partial \eta(t_w)} = \frac{df_s\{t_w - t_0\}}{dN} = -f_s'\{t_w - t_0\}\frac{dt_0}{dN} \tag{A1}$$

On the other hand, in general, queue starting time t_0 is determined so that $p(t_1)=0$ when the queue vanishes at time t_1. From Eq.(20),

$$p(t_1) = \int_{t_0}^{t_1} f_s'\{s(t)\}dt = f_s\{t_w - t_0\} - f_s\{t_w - t_1\} = 0 \tag{A2}$$

Plugging t_1-t_0=N/μ into the above,

$$f_s\{t_w - t_0\} = f_s\{(t_w - t_0) - \frac{N}{\mu}\} \tag{A3}$$

The differentiation of Eq.(A3) yields

$$\frac{dt_0}{dN} = -\frac{1}{\mu}[1 - \frac{f_s'\{t_w - t_0\}}{f_s'\{t_w - t_1\}}]^{-1} \tag{A4}$$

Thus, from Eq.(A1),

$$\frac{\partial p(u)}{\partial \eta(tw)} = \frac{1}{\mu}[\frac{1}{f_s'\{t_w - t_0\}} - \frac{1}{f_s'\{t_w - t_1\}}]^{-1} \tag{A5}$$

$$MC(t_w) = p(t_w) + \sum_u \frac{\partial p(u)}{\partial \eta(t_w)} \eta(u) = f_s\{t_w - t_0\} + \frac{N}{\mu}[\frac{1}{f_s'\{t_w - t_0\}} - \frac{1}{f_s'\{t_w - t_1\}}]^{-1},$$

$$\text{if } \; w(t_w) > 0 \; \text{ or } \; \lambda(t_w) \geq \mu$$

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