The departure-time choice equilibrium of the corridor problem with discrete multiple bottlenecks

modeling, solvability, and uniqueness

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Bottleneck and Traffic Congestion

We often encounter the traffic congestion (traffic jam) at the entrance of a bottleneck (BN).

e.g. bridge, interchange, construction, etc.

The traffic capacity of the BN is limited. There happens the congestion when the traffic flow concentrates over the BN capacity.

Economical loss due to the congestion is quite a bit.

It is important to analyze the congestion due to the bottlenecks.
Existing BN and congestion researches

[Single BN model]

- Definition of departure-time choice equilibrium [Vickrey 1969]
- More detailed equilibrium model [Hendrickson et. al., 1981]
- Existence and uniqueness of equilibrium [Smith, 1984] [Daganzo, 1985]

[Multiple BN model]

- Extension to 2 tandem BNs with equilibrium analyses [Kuwahara, 1990], [Arnott et al. 1993]
- Extension to N tandem BNs [Arnott and De Palma, 2011]

Analyses of equilibrium (existence/uniqueness) is not sufficient.

We reformulate the departure time choice equilibrium problem with N tandem BNs as a linear/nonlinear complementarity problem. Then, we study its existence and uniqueness property.

Note: This equilibrium model is different from the well-known Wordrop equilibrium, which is the time-independent equilibrium model based on the route choice.
Consider many-to-one OD pairs with \( N \) tandem bottlenecks. (morning rush model)

- There exist \( Q_i \) users (commuters / vehicles) in each Zone \( i \). \((i = 1, 2, \ldots, N)\)
- Every morning, all users commute to the same CBD (central business district).
- Users in Zone \( i \) go through the downstream BNs \( i, i - 1, \ldots, 2, 1 \).
- First-In-First-Out (FIFO) is assumed.

Notice the correspondence between zone numbers and BN numbers.
Choice of departure time

Each user chooses the departure time to minimize his/her travel cost.

$t$: departure time ($0 \leq t \leq T$)

$\lambda_i(t)$: density of users living in Zone $i$ who choose $t$

$Q_i$: Number of users in Zone $i$ (traffic demand)

Notice: At first, time and number of users are supposed to be continuous. (We discretize them later.)

$T$ is sufficiently large number.

This area equals the number of users departing in $[t_1, t_2]$

$$Q_i = \int_{t_1}^{t_2} \lambda_i(t) \, dt$$
Departure Time Choice Equilibrium (DTCE)

The (traffic) flow pattern satisfying the following 3 conditions (a)-(c) is called Departure Time Choice Equilibrium (DTCE).

(a) Queuing Condition
How is the queue generated at BN? How long do we wait at the queue?

(b) Departure Time Choice Policy
How does each user choose his/her departure time?

(c) Flow Conservation Law
(a) Queuing Condition
(single BN case)

\[ A(t) \]
\[ D(t) \]
\[ E(t) \]

**gradient** = \( \mu \)
(capacity of BN)

\[ t : \text{time} \quad (0 \leq t \leq T) \]

\( A(t) : \text{cumulative num. of users arriving at the entrance of BN} \)

\( D(t) : \text{cumulative num. of users departing from BN} \)

\( \mu : \text{capacity of BN} \quad (\mu > 0) \)
(maximum number of users who can pass through the BN per unit time)

\[ E(t) = A(t) - D(t) : \text{num. of users queuing at BN} \]

\[ d(t) = E(t)/\mu : \text{waiting time in the queue} \]

\[ \dot{D}(t) = \begin{cases} 
\mu & (d(t) > 0) \\
\min(\dot{A}(t), \mu) & (d(t) = 0)
\end{cases} \]

\( \dot{D}(t) : \text{departure rate} \)

Note: Physical size of queue or vehicles are ignored.
(a) **Queuing Condition**

*(single BN case)*

\[
\dot{D}(t) = \begin{cases} 
\mu & (d(t) > 0) \\
\min(\dot{A}(t), \mu) & (d(t) = 0)
\end{cases}
\]

\[
E(t) = A(t) - D(t) \quad d(t) = E(t)/\mu
\]

\[
\dot{d}(t) = (\dot{A}(t) - \dot{D}(t))/\mu
\]

\[
= \begin{cases} 
[\dot{A}(t)/\mu] - 1 & (d(t) > 0) \\
\max(0, [\dot{A}(t)/\mu] - 1) & (d(t) = 0)
\end{cases}
\]

\[
0 \leq d(t) \perp \dot{d}(t) - ([\dot{A}(t)/\mu] - 1) \geq 0 \quad (\forall t \in [0, T])
\]

The waiting time \(d(t)\) has to satisfy the above complementarity condition.

*(Here, \(0 \leq \alpha \perp \beta \geq 0\) denotes \(\alpha \geq 0, \beta \geq 0, \alpha \beta = 0\). )

We want to extend this formulation to the multiple BN model.
Extention to multiple BN case

(based on the existing time coordinate system)

If we directly inherit the time coordinate system from the single BN case, then the time variables becomes quite nested. impossible to analyze!

\[ \pi_i(t_i) \]
\[ d_i(t_i) \]
\[ c_i \]

\[ t_{i-1} = t_i + (d_i(t_i) + c_i) \]
\[ t_{i-2} = t_{i-1} + (d_{i-1}(t_{i-1}) + c_{i-1}) \]
\[ \ldots \]
\[ t_0 = t_i + \pi_i(t_i) \]

\( t_i \): arrival time at BN \( i \)  \( \pi_i(t_i) \): total travel time from Zone \( i \) to CBD
\( c_i \): free travel time from BN \( i \) to BN \( i-1 \)  \( d_i(t_i) \): waiting time at BN \( i \)

In stead of using actual time \( t \), we introduce a new time coordinate system based on CBD arrival time \( s \).
Time coordinate system based on CBD arrival

\[ s \in \mathcal{S} = [S, \bar{S}] \]

\( \tau_i(s) \): arrival time at the entrance of BN \( i \)

(The user chooses his/her departure time so that the CBD arrival time is \( s \).

(Due to FIFO, \( \tau_i(s) \) do not depend on the zone in which user lives.)

**Single BN case**

\[
0 \leq d(t) \perp \dot{d}(t) - (\lfloor \dot{A}(t)/\mu \rfloor - 1) \geq 0
\]

\( A(t) \): cumulative num. of users arriving at the entrance of BN

\( d(t) \): waiting time in the queue

**Multiple BNs** (complementarity condition for BN \( i \))

\[
0 \leq d_i(t) \perp \dot{d}_i(t) - (\lfloor \dot{A}_i(t)/\mu_i \rfloor - 1) \geq 0
\]

\( t := \tau_i(s) \)

\[
0 \leq d_i(\tau_i(s)) \perp \dot{d}_i(\tau_i(s)) - (\lfloor \dot{A}_i(\tau_i(s))/\mu_i \rfloor - 1) \geq 0
\]

\( \mu_i \): capacity of BN \( i \)

Note: The capacity of each BN can be different. \( (\mu_i \neq \mu_j \text{ for } i \neq j) \)
Time coordinate system based on CBD arrival

\[ 0 \leq d_i(\tau_i(s)) - \left( \left[ \hat{A}_i(\tau_i(s))/\mu_i \right] - 1 \right) \geq 0 \]

\((\cdot) := \frac{d(\cdot)}{ds}\)

Note: \(\tilde{\tau}_i(s) > 0\) due to FIFO assumption

\[ 0 \leq d_i(\tau_i(s)) - \dot{d}_i(\tau_i(s)) \cdot \tilde{\tau}_i(s) - \left( \left[ \hat{A}_i(\tau_i(s)) \cdot \tilde{\tau}_i(s)/\mu_i \right] - \tilde{\tau}_i(s) \right) \geq 0 \]

\(w_i(s) := d_i(\tau_i(s))\) : waiting time at BN \(i\) (w.r.t. new coordinate system \(s\))

\(Y_i(s) := A_i(\tau_i(s))\) : cumulative number of users arriving at BN \(i\) (w.r.t. new coordinate system \(s\))

\[ 0 \leq w_i(s) - \ddot{w}_i(s) - \left( \left[ \ddot{Y}_i(s)/\mu_i \right] - \tilde{\tau}_i(s) \right) \geq 0 \]

\[ 0 \leq w_i(s) - \mu_i(\ddot{w}_i(s) + \tilde{\tau}_i(s)) - \dot{Y}_i(s) \geq 0 \]

We have derived the queuing condition based on the CBD arrival time \(s\).
Departure Time Choice Equilibrium (DTCE)

The (traffic) flow pattern satisfying the following 3 conditions (a)-(c) is called Departure Time Choice Equilibrium (DTCE).

(a) Queuing Condition

\[ 0 \leq w_i(s) \perp \mu_i(\tilde{w}_i(s) + \tilde{r}_i(s)) - \tilde{Y}_i(s) \geq 0 \]

(b) Departure Time Choice Policy

How does each user choose his/her departure time?

(c) Flow Conservation Law

We say “departure time choice” in the sense that each driver chooses the departure time so that the CBD arrival time becomes \( s \).
Users’ Time Choice

Each user chooses CBD arrival time \( s \)

\[ s : \text{CBD arrival time} \quad (s \in S = [S, \overline{S}]) \]

\[ q_i(s) : \text{density of users living in Zone } i \text{ who choose } s \]

\[ Q_i : \text{number of users in Zone } i \]

\[ Q_i = \int_S q_i(s) ds \]

This area equals the number of users in Zone \( i \) arriving at CBD in the interval \([s_1, s_2]\)
Users’ Time Choice

User in Zone $i$ chooses his/her CBD arrival time to minimize the cost function.

$$\text{cost } u_i(s) := \text{total travel time } s - \tau_i(s) + \text{schedule delay penalty } p(s)$$

$\tau_i(s)$: arrival time at BN $i$

($= \text{departure time of the user living in Zone } i$)

**schedule delay penalty**

$$p(s) := \begin{cases}  -\alpha(s - t_w) & (s < t_w) \\ \beta(s - t_w) & (s \geq t_w) \end{cases}$$

where $0 < \alpha < 1 < \beta$

$t_w$: desired arrival time

In this study, we assume that all users in every zone have the same schedule delay penalty function.
Time Choice Policy A (min type)

[Policy A] min type (deterministic time choice)

Each user in Zone $i$ chooses time $s$ such that the cost $u_i(s)$ is minimum.

\[
\begin{align*}
q_i(s) > 0 &\implies u_i(s) = \min_{s' \in [s, S]} u_i(s') \\
q_i(s) = 0 &\iff u_i(s) > \min_{s' \in [s, S]} u_i(s') \\
(\mu_i \neq \mu_j \text{ for } i \neq j)
\end{align*}
\]

\[
0 \leq q_i(s) \perp u_i(s) - \rho_i \geq 0
\]

$\rho_i$: minimum cost for users in Zone $i$

\[
\left(\rho_i = \min_{s' \in [s, S]} u_i(s')\right)
\]

\[u_i(s) = [s - \tau_i(s)] + p(s)\]
Time Choice Policy B (logit type)

[Policy B] logit type (probabilistic time choice)

Each user in Zone $i$ chooses the CBD arrival time $s$ according to the following 
probabilistic density function $P_i(s)$.

$$P_i(s) = \frac{\exp(-\theta u_i(s))}{\int_S \exp(-\theta u_i(s)) ds}$$

$\theta > 0$: parameter

$P_i$ is the so-called logit function.

If the user has only 2 choices $s_1$ and $s_2$, then $P_i$ is as follows.

\[P_i(s_1)\]
Time Choice Policy B (logit type)

[Policy B] logit type (probabilistic time choice)

Each user in Zone $i$ chooses the CBD arrival time $s$ according to the following probabilistic density function $P_i(s)$.

$$
\int_S P_i(s) \, ds = 1
$$

$$
P_i(s) = \frac{\exp(-\theta u_i(s))}{\int_S \exp(-\theta u_i(s)) \, ds}
$$

$\theta > 0$: parameter

Density of users $q_i(s)$ choosing CBD arrival time $s$

$$
q_i(s) = Q_i P_i(s)
= \frac{Q_i \exp(-\theta u_i(s))}{\int_S \exp(-\theta u_i(s)) \, ds}
$$

$Q_i$: number of users living in Zone $i$

Note: When $\theta \to +\infty$, the logit type approaches to the min type.
Departure Time Choice Equilibrium (DTCE)

The (traffic) flow pattern satisfying the following 3 conditions (a)-(c) is called Departure Time Choice Equilibrium (DTCE).

(a) Queuing Condition

\[ 0 \leq w_i(s) \perp \mu_i(\tilde{w}_i(s) + \tilde{\tau}_i(s)) - \tilde{Y}_i(s) \geq 0 \]

(b) Departure Time Choice Policy

\[ 0 \leq q_i(s) \perp u_i(s) - \rho_i \geq 0 \]  
(min type)

or

\[ q_i(s) = \frac{Q_i \exp(-\theta u_i(s))}{\int_{S} \exp(-\theta u_i(s))ds} \]  
(logit type)

(c) Flow Conservation Law
(c) Flow Conservation Law

\[ \int_S q_i(s) \, ds = Q_i \]

\[ \iff 0 \leq \rho_i \perp \int_S q_i(s) - Q_i \geq 0 \]

(\therefore \rho_i > 0)

When the time choice policy is logit type, this condition holds automatically.

**logit case**

\[ q_i(s) = Q_i P_i(s) \]

\[ P_i(s) \text{ is a probability density function} \]
Departure Time Choice Equilibrium (DTCE)

(a) Queuing Condition

\[ 0 \leq w_i(s) \perp \mu_i(\bar{w}_i(s) + \bar{r}_i(s)) - \bar{Y}_i(s) \geq 0 \]

(b) Departure Time Choice Policy

\[ 0 \leq q_i(s) \perp u_i(s) - \rho_i \geq 0 \quad \text{(min type)} \]

or

\[ q_i(s) = \frac{Q_i \exp(-\theta u_i(s))}{\int_S \exp(-\theta u_i(s)) \, ds} \quad \text{(logit type)} \]

(c) Flow Conservation Law

\[ 0 \leq \rho_i \perp \int_S q_i(s) - Q_i \geq 0 \quad \text{(min type)} \]
Infinite dimensional complementarity reformulation

Combining three conditions (a)-(c), we obtain the following infinite dimensional complementarity reformulation

\[
\begin{align*}
\min \text{ type} & \quad \rightarrow \quad \text{infinite dimensional LCP} \quad (\text{variable: } w, q, \rho) \\
0 \leq w_i(s) & \perp \mu_i - \mu_i \sum_{j=1}^{i-1} \bar{w}_i(s) - \sum_{j=i}^{N} q_j(s) \geq 0 \\
0 \leq q_i(s) & \perp \sum_{j=1}^{i} (w_j(s) + c_j) + p(s) - \rho_i \geq 0 \\
0 \leq \rho_i & \perp \int_{S} q_i(s) - Q_i \geq 0 \quad (i = 1, \ldots, N, s \in S = [\underline{S}, \bar{S}])
\end{align*}
\]

\[
\begin{align*}
\logit \text{ type} & \quad \rightarrow \quad \text{infinite dimensional NCP} \quad (\text{variable: } w, q) \\
0 \leq w_i(s) & \perp \mu_i - \mu_i \sum_{j=1}^{i-1} \bar{w}_i(s) - \sum_{j=i}^{N} q_j(s) \geq 0 \\
0 \leq q_i(s) & \perp q_i(s) - \frac{Q_i \exp(-\theta \sum_{j=1}^{i} (w_j(s) + c_j) + p(s))}{\int_{S} \exp(-\theta \sum_{j=1}^{i} (w_j(s) + c_j) + p(s)) \, ds} \geq 0 \\
& \quad (i = 1, \ldots, N, s \in S = [\underline{S}, \bar{S}])
\end{align*}
\]

Note: Since \( \tau_i(s) = s - \sum_{j=1}^{i} (w_j(s) + c_j) \), \( \tilde{Y}_i(s) = \sum_{j=i}^{N} q_j(s) \), and \( u_i(s) = [s - \tau_i(s)] + p(s) \), we eliminated those variables.
Finite LCP reformulation by time descretization

\[
\begin{align*}
0 \leq w_i(s) & \perp \mu_i - \mu_i \sum_{j=1}^{i-1} \tilde{w}_i(s) - \sum_{j=i}^{N} q_j(s) \geq 0 \\
0 \leq q_i(s) & \perp \sum_{j=1}^{i} (w_j(s) + c_j) + p(s) - \rho_i \geq 0 \\
0 \leq \rho_i & \perp \int_S q_i(s) - Q_i \geq 0 \quad (i = 1, \ldots, N, s \in S = [\underline{S}, \overline{S}])
\end{align*}
\]

Time descretization:
\[
[S, \overline{S}] \rightarrow \{S + \Delta s, S + 2\Delta s, \ldots, S + K\Delta s\} \quad \text{with} \quad \Delta s = (\overline{S} - \underline{S}) / K
\]

\([\text{DUE-M(B)-LCP}] \quad \text{Find} \quad X \equiv [q, w, \rho]^T \quad \text{such that} \quad 0 \leq X \perp F(X) : MX + b \geq 0,
\]

where \( M \in \mathbb{R}^{(2K+1)N \times (2K+1)N} \) and \( b \in \mathbb{R}^{(2K+1)N} \) are defined as
\[
M = \begin{bmatrix}
I_K \otimes L & -I_K \otimes I \\
-I_K \otimes L^T & \Delta_K \otimes C[I - L]
\end{bmatrix}, \quad b = \begin{bmatrix}
p + I_K \otimes Lc \\
I_K \otimes C1 \\
-\mathbf{Q}
\end{bmatrix}
\]

where \( q = (q_i(S + \Delta s), \ldots, q_i(S + K\Delta s))_i^{N} \in \mathbb{R}^{NK}, \quad w \in \mathbb{R}^{NK}, \quad \rho \in \mathbb{R}^{N} \)

\[
C = \text{diag}(\mu_i) \quad L = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
\vdots & \vdots & \vdots \\
1 & 1 & 1
\end{bmatrix} \quad \Delta_K = \begin{bmatrix}
1 & -1 & 1 & \cdots & 1 \\
-1 & 1 & -1 & \cdots & -1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & -1 & 1 & \cdots & -1
\end{bmatrix} \quad \Delta s := 1
\]
Finite LCP reformulation by time discretization

Logit type

\[ 0 \leq w_i(s) \perp \mu_i - \mu_i \sum_{j=1}^{i-1} \bar{w}_i(s) - \sum_{j=i}^{N} q_j(s) \geq 0 \]

\[ q_i(s) = \frac{Q_i \exp(-\theta \sum_{j=1}^{i} (w_j(s) + c_j) + p(s))}{\int_S \exp(-\theta \sum_{j=1}^{i} (w_j(s) + c_j) + p(s)) ds} \]

\[ (i = 1, \ldots, N, s \in S = [S, \overline{S}]) \]

Time discretization:

\([S, \overline{S}] \rightarrow \{S + \Delta s, S + 2\Delta s, \ldots, S + K\Delta s\} \text{ with } \Delta s = (\overline{S} - S)/K\]

[DUE-M-MCP]: Find \(X \equiv [q, w]^T \in \mathbb{R}^{2NK}\) and \(\Delta \tau_N\) such that

\[ 0 \leq X \perp G_S(q, w) - G_D(w) + G_\tau(\Delta \tau_N) \geq 0, \]

\[ \Delta \tau_N + (\Delta w_N + \cdots + \Delta w_1) = 1_K \]

where

\[ G_S \equiv \begin{bmatrix} I \otimes I_K & 0 \\ -L^T \otimes I_K & CL^T \otimes \Delta_K \end{bmatrix} \begin{bmatrix} q \\ w \end{bmatrix}, \quad G_D \equiv \begin{bmatrix} QP(w) \\ 0 \end{bmatrix}, \quad G_\tau \equiv \begin{bmatrix} 0 \\ C1 \otimes \Delta \tau_N \end{bmatrix} \]
Evening rush model (one-to-many)

We can also consider the evening-rush model.

Basically, the complementarity model can be obtained by re-defining the time coordinate system in a backward direction.
Evening rush model (one-to-many)

Evening rush model (min type)

\[[\text{DUE-E-LCP}]\quad \text{Find } X \equiv [q, w, \rho]^T \text{ such that } 0 \leq X \perp F(X) \equiv MX + b \geq 0,\]

where \( M \in \mathbb{R}^{(2K+1)N \times (2K+1)N} \) and \( b \in \mathbb{R}^{(2K+1)N} \) are defined as

\[
M = \begin{bmatrix}
I_K \otimes L & -1_K \otimes I \\
-I_K \otimes L^T & \Delta_K \otimes CL \\
1^T_K \otimes I & -Q
\end{bmatrix}, \quad b = \begin{bmatrix}
p + I_K \otimes Lc \\
I_K \otimes C1 \\
-Q
\end{bmatrix},
\]

\[
L = \begin{bmatrix}
1 & 1 & 1 \\
\vdots & \ddots & \ddots \\
1 & \cdots & 1 & 1
\end{bmatrix}, \quad \Delta_K = \begin{bmatrix}
1 & -1 & 1 & \cdots & 1 & 1 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
-1 & \cdots & \cdots & \cdots & \cdots & 1
\end{bmatrix}
\]

Evening rush model (logit type)

\[[\text{DUE-E-NCP}]\quad \text{Find } X \equiv [q, w]^T \in \mathbb{R}^{2NK} \text{ such that }\]

\[
0 \leq X \perp G(X) \equiv \begin{bmatrix}
I \otimes I_K & 0 \\
-L^T \otimes I_K & CL \otimes \Delta_K
\end{bmatrix} \begin{bmatrix}
q \\
w
\end{bmatrix} + \begin{bmatrix}
-QP(w) \\
C1 \otimes I_K
\end{bmatrix} \geq 0.
\]
Existence of equilibrium

For each model, we have shown the existence of the departure time choice equilibrium.

Theorem

There exist at least one equilibrium in either case:

(i) Morning rush model (min type)
(ii) Morning rush model (logit type)
(iii) Evening rush model (min type)
(iv) Evening rush model (logit type)
Overview of the proof (morning/min case)

\[
\begin{align*}
0 \leq w & \perp [\Delta_K \otimes D_\mu(I - L)]w - [I_K \otimes L^\top]q + [I_K \otimes D_\mu 1] \geq 0 \quad (a) \\
0 \leq q & \perp [I_K \otimes L]w - [1_K \otimes I]p + [p \otimes 1 + 1_K \otimes Lc] \geq 0 \quad (b) \\
0 \leq \rho & \perp [1_K \otimes I]q - Q \geq 0 \quad (c)
\end{align*}
\]

In (a), \(w\) can be expressed as a function of \(q\).
\[
\exists W : \mathbb{R}^{NK} \rightarrow \mathbb{R}^{NK}, \text{ s.t. } w = W(q)
\]

In (b) and (c), \(q\) is expressed as a set-valued mapping of \(w\).
\[
\exists \tilde{q} : \mathbb{R}^{NK} \rightarrow 2^{\mathbb{R}^{NK}}, \text{ s.t. } q \in \tilde{q}(w)
\]

\(\rho\) is eliminated.

\[\iff\] If the fixed point problem \(q \in \tilde{q} \circ W(q)\) has a solution, then the above LCP also has a solution.

- \(\tilde{q} \circ W\) is upper semi-continuous, and any image is nonempty, closed and convex.
- The domain \(\Omega\) of \(\tilde{q} \circ W\) is nonempty, compact, and satisfies \(\tilde{q} \circ W(\Omega) \subseteq \Omega\)

\[\iff\] We can apply Kakutani’s fixed-point theorem.
Overview of the proof (other cases)

(Evening rush model: min type)

[DUE-E-LCP]: Find $X = [q, w, \rho]^T$ such that $0 \leq X \perp F(X) = MX + b \geq 0$, where $M \in \mathbb{R}^{(2K+1)N \times (2K+1)N}$ and $b \in \mathbb{R}^{(2K+1)N}$ are defined as:

$$
M = \begin{bmatrix}
I_K \otimes L & -I_K \otimes I \\
-I_K \otimes L^T & \Lambda_K \otimes CL \\
1_K \otimes I & -Q
\end{bmatrix},
$$

$$
b = \begin{bmatrix}
p + I_K \otimes Lc \\
I_K \otimes C1 \\
-Q
\end{bmatrix}.
$$

Proved by Kakutani's theorem

(Morning rush model: logit type)

[DUE-M-MCP]: Find $X = [q, w]^T \in \mathbb{R}^{2NK}$ and $\Delta \tau_N$ such that:

$$
0 \leq X \perp G_S(q, w) - G_D(w) + G_\tau(\Delta \tau_N) \geq 0,
$$

$$
\Delta \tau_N + (\Delta w_N + \cdots + \Delta w_1) = 1_K
$$

where

$$
G_S = \begin{bmatrix}
I \otimes I_K & 0 \\
-L^T \otimes I_K & CL^T \otimes \Lambda_K
\end{bmatrix},
G_D = \begin{bmatrix}
QP(w) \\
0
\end{bmatrix},
G_\tau = \begin{bmatrix}
0 \\
C1 \otimes \Delta \tau_N
\end{bmatrix}.
$$

(Evening rush model: logit type)

[DUE-E-NCP]: Find $X = [q, w]^T \in \mathbb{R}^{2NK}$ such that:

$$
0 \leq X \perp G(X) = \begin{bmatrix}
I \otimes I_K & 0 \\
-L^T \otimes I_K & CL \otimes \Lambda_K
\end{bmatrix} \begin{bmatrix}
q \\
w
\end{bmatrix} + \begin{bmatrix}
-QP(w) \\
C1 \otimes I_K
\end{bmatrix} \geq 0.
$$

Proved by Brower's theorem
Uniqueness of equilibrium

Evening rush model (logit type)

Proved. (We analyzed the determinant of Jacobian appearing in the NCP reformulation.)

Morning rush model (logit type)

Proved under a certain assumption.

Evening rush model (min type)

Morning rush model (min type)

Not Proved.

• We have confirmed experimentally that the uniqueness is satisfied with respect to $w$.
• With respect to $q$, we have a counterexample such that the uniqueness does not hold.
Numerical example (morning rush model: logit type)

#BN: N=3, time #disc.: K=60, BN capa: μ = (30, 20, 10), # users Q = (100, 200, 300)

Red: cum. arrival to BN, Blue: cum. departure from BN

BN3 (upstream)  BN2 (middle stream)  BN1 (downstream)

Total of all zones

Zone 1 residents

Zone 2 residents

Zone 3 residents

First row: Aggregate cumulative curves
Second row: Disaggregate cumulative curves for users with origin 1
Third row: Disaggregate cumulative curves for users with origin 2
Fourth row: Disaggregate cumulative curves for users with origin 3
Numerical example (evening rush model: logit type)

#BN: N=3, time #disc.: K=60, BN capa: \( \mu = (30, 20, 10) \), # users Q=(100, 200, 300)

Red: cum. arrival to BN, Blue: cum. departure from BN

First row: Aggregate cumulative curves
Second row: Disaggregate cumulative curves for users with destination 1
Third row: Disaggregate cumulative curves for users with destination 2
Fourth row: Disaggregate cumulative curves for users with destination 3

Total of all zones

Zone 1 residents

Zone 2 residents

Zone 3 residents

Desired departure time
The 21st International Symposium on Transportation and Traffic Theory
5 - 7 August, 2015, Kobe, Japan

Accompanying Persons' Programme
Please see this PDF file for the details of the accompanying person programme. If you are interested please contact via the e-mail adress in the file.

Registration Deadline: Jul 22, 2015
The registration site opens till 22nd July. Please register to the symposium at your earliest convenience.

Welcome
The 21st International Symposium on Transportation and Traffic Theory (ISTTT) will be held at Kobe International Conference Center, Kobe, Japan, from 5th August to 7th August, 2015. The 21st Symposium will be organized by the local organizing committee of Japan. The series of ISTTT (see http://www.isttt.net/) is the premier gathering for the world’s leading transportation and traffic theorists, and for those who are interested in contributing to or gaining a deeper understanding of the field. The symposium covers all scientific aspects of transportation and traffic, spanning all modes of transport, including freight, air, and maritime modes, as well as private and public transport.

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- Michel Bierlaire: Ecole Polytechnique Federale de Lausanne, Switzerland
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- ISTTT is the top conference in the area of transportation theory.
- This talk will be also presented by my co-author, and published in the Special Issue of Transportation Research Part B: Methodological.